Prince Sultan University

Department of Mathematical Sciences Major I Exam

Semester II, 2014 SPRING (132) 10th March, 2014

MATH 111 – CALCULUS I

Time Allowed : 90 minutes Maximum Points: 60 points

Name of the student: _____

ID number

| Dr. Nasreddine Megrez | Mr. Abed Zargar | Mr. Khaled Naseralla |
|-----------------------|-----------------|----------------------|
| Section 219 | Section 218 | Section 217 |
| 11 12 | 10 11 | 8 9 |

Important Instructions

:

- You may use a **SCIENTIFIC CALCULATOR** that does **NOT** have **GRAPHING** capabilities.
- You may **NOT** borrow a calculator from anyone.
- Answer **ALL** the questions.
- Show all the key steps of your work in the space provided for each question. Please indicate your **FINAL** answers clearly.
- You may use the **BACK** of the pages for extra space, but be sure to indicate that on the page with the problem.
- There should be **NO** talking during the exam.
- This exam has **13** problems, some with several parts. Make sure your paper has all these problems.

| Question | Maximum Points | Points Earned |
|-----------|-------------------|------------------|
| 1 , 2 , 3 | 12 | |
| 4 , 5 , 6 | 15 | |
| 7,8,9 | 16 | |
| 10 , 11 | 26 | |
| 12 , 13 | 11 | |
| | | |
| Total | 80 | |

<u>Q.1 (4 points)</u>: Given that $f(x) = \frac{1}{x^2}$ find the Difference Quotient, $\frac{f(x+h) - f(x)}{h}$.

<u>Q.2 (3 points)</u>: Find the Domain of $f(x) = \frac{3x-9}{\sqrt{x^2 + x - 6}}$

<u>Q.3 (5 points)</u>: Write the following function in piecewise form, without absolute value bars f(x) = |x+3|+2|x-2|.

<u>Q.4 (4 points)</u>: Find an exponential function of the form $y = Ca^x$ using the given graph.



<u>Q.5 (8 points)</u>: (i) Use transformations to sketch the graph of $f(x) = -\frac{1}{2}e^{(x-1)} + 3$. Show the location of the asymptote and the *x* or *y* intercepts, if any. (Without using a table of values)

- (ii) State the domain and range of f.
- (iii) Find the inverse of *f*, and state its domain and range.



- (i) f(g(3))
- (ii) $(g \circ g)(6)$

(iii) $(f \circ f)(-1)$



<u>Q.7 (4 points)</u>: Solve the equation $e^{3x+1} = 3e^{x-2}$

<u>Q.8 (8 points)</u>: Use the following graph to answer the following questions

- У зŤ a) $\lim_{x \to -1^+} g(x) =$ 2. b) $\lim_{x \to -1^{-}} g(x) =$ 1. c) $\lim_{x \to -1} g(x) =$ -2 -1 ż -3 1 d) $\lim_{x \to 1} f(x) =$ -1 -21 e) $\lim_{x \to 2^{-}} g(x) =$
 - f) $\lim_{x \to -\infty} g(x) =$
 - g) Where is f continuous only from the right?
 - h) Is f continuous at x = 2? Justify your answer.

<u>Q.9 (4 points)</u>: Show that $\lim_{x \to \infty} \frac{1}{x^2} \sin(x) = 0$

<u>Q.10 (22 points):</u> Evaluate the limit, if it exists.

(i)
$$\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$$

(ii)
$$\lim_{y \to 0} \frac{2 - \sqrt{4 - y}}{y}$$

(iii)
$$\lim_{x \to 2} \frac{x^2 + 1}{x - 2}$$

(iv)
$$\lim_{x \to 3} \frac{x^2 - 9}{|x - 3|}$$

Show your detailed solutions

(v)
$$\lim_{x \to \infty} \frac{x^3 - 3x + 1}{1 - 4x^3}$$

(vi)
$$\lim_{x \to -\infty} \frac{1 - e^{-2x}}{1 + 2e^{-2x}}$$

(vii)
$$\lim_{x\to\infty} \left(\sqrt{9x^2 + x} - 3x\right)$$

<u>Q.11 (4 points)</u>: Show that the equation: $x^3 - 15x = -1$ has a root in the interval [3,4]

Q.12 (6 points): Determine all vertical and horizontal asymptotes (if any) for the following function.

$$f(x) = \frac{2\sqrt{x^2 + 4}}{x - 3}$$

<u>**Q.13 (5 points)**</u>: Discuss the continuity of the following function. Find the value of k, if possible, that will make the function continuous everywhere.

$$f(x) = \begin{cases} ke^{3x} & x \ge -1 \\ x^3 + x + 1 & x < -1 \end{cases}$$