Prince Sultan University
Department of Mathematical Sciences
Major II Exam
Semester II, 2015   SPRING (142)
20th April, 2015

MATH 111 – CALCULUS I

Time Allowed   : 90 minutes
Maximum Points: 80 points

Name of the student: _____

ID number   : _____

Dr. Kamal Abodayeh	Dr. Nabil Mlaiki	Mr. Khaled Naseralla
Section 223	Section 225	Section 224
8 ----- 9	11 ----- 12	10 ----- 11

Important Instructions

- You may use a **SCIENTIFIC CALCULATOR** that does **NOT** have **GRAPHING** capabilities.

- You may **NOT** borrow a calculator from anyone.

- Answer **ALL** the questions.

- Show all the key steps of your work in the space provided for each question. Please indicate your **FINAL** answers clearly.

- You may use the **BACK** of the pages for extra space, but be sure to indicate that on the page with the problem.

- There should be **NO** talking during the exam.

- This exam has **10** problems, some with several parts. Make sure your paper has all these problems.

Question	Maximum Points	Points Earned
1	23	
2 , 3 , 4	17	
5	12	
6 , 7 , 8	16	
9 , 10	12	
Total	**80**	

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Q.1 (23 points):   Find the derivative and **simplify** your answer.

a) $y = \tan^{-1}\left(\tanh(3x)\right)$

b) $y = 3^{x\ln(x)}$

c) $y = \dfrac{x^2\sqrt{x^4-1}}{(2x+1)e^x}$

d) $f(x) = \cosh^2\left(\dfrac{x}{x+1}\right)$

e) $g(x) = \left(\sin^{-1} \left(5x^2 \right) \right)^3$

f) $3x^2 + 3 = \ln\left(5xy^2 \right)$

g) $y = x^{\sec(x)}$

Q.2 (5 points): Let f and g be functions such that $f(1) = -1$, $f'(1) = 3$, and $g'(-1) = -4$.

Let $F(x) = 2(f(x))^2 - g(f(x))$. Find $F'(1)$.

Q.3 (6 points): Find the equation of the tangent lines to the curve:

$$x^2 + 4xy + y^2 = 13 \quad \text{at the point } (2,1)$$

Q.4 (6 points): Find the equation of the **normal** line to the curve of $y = \dfrac{x^2 + 1}{2\tan(x) + 1}$ at the point $(0,1)$.

Q.5 (12 points): Prove the identity:

a) $\cosh(x) - \sinh(x) = e^x$

b) $\coth^2(x) - 1 = \csc h^2(x)$

c) Show that $\tanh^{-1}(x) = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$, $-1 < x < 1$

Q.6 (5 points): Show that the derivative of $y = x\cosh^{-1}\left(\dfrac{x}{3}\right) - \sqrt{x^2 - 9}$ is $y' = \cosh^{-1}\left(\dfrac{x}{3}\right)$

Q.7 (5 points): Find $\dfrac{d^{43}}{dx^{43}}\left(x.\cos(x)\right)$

Q.8 (6 points): Two cars start moving from the same point. One travels north at 50 km/h and the other travels west at 70 km/h. At what rate is the distance between the cars increasing 2 hours later?

Q.9 (8 points): Find the limit:

a) $\lim\limits_{x\to 0} \dfrac{\tan^2(5x) + \sin^2(3x)}{x^2}$

b) $\lim\limits_{x\to -3} \dfrac{\sin(x+3)}{x^2 + 8x + 15}$

Q.10 (4 points): If a snow ball melts at a rate of $0.5 \text{ cm}^3/\min$. Find the rate at which the radius decreases when the radius is 5 cm.