

Prince Sultan University
Department of Mathematical Sciences
Major I Exam
Semester II, 2005 Spring (042)
23rd March, 2005
MATH 111 – CALCULUS I

Time Allowed : 90 minutes

Maximum Points: 100 points

Name of the student : _____

ID number : _____

Section : _____

For All The Students:

- Answer all the questions.
- This exam consists of **a total of 7 pages and 14 questions.**
- Show your working in the space provided for each question.
- Show all the key steps of your work.
- Scientific, non-programmable calculators are allowed.

Question	Maximum score	Your Score
Q.1	6	
Q.2	5	
Q.3	5	
Q.4	5	
Q.5	6	
Q.6	6	
Q.7	8	
Q.8	21	
Q.9	8	
Q.10	5	
Q.11	5	
Q.12	5	
Q.13	10	
Q.14	5	
Total	100	

Q.1 : Use the equation : $y = x^2 - 6x + 8$ to answer the following questions: (6 points)

a) For what value at x is $y = 15$?

b) For what value at x is $y \leq 15$?

c) What is the minimum value of y ?

Q.2: Find the slope-intercept form of the equation of the line perpendicular (5 points)
To: $3x + 2y = 5$ and passes through the point $(0, 3)$

Q.3: Find an equation for the family of lines passing through the point $(-2, 3)$. (5 points)
Sketch two of the lines.

Q.4: State the geometric property common to all the lines in the family: (5 points)

$$Ax - 2y + 6 = 0$$

Q.5:

Find the natural domain of each of the following functions:

(6 points)

a) $f(x) = \sqrt{\frac{8x-4}{x-3}}$

b) $g(x) = \frac{x^2}{x^2-3x+2}$

Q.6:

Express the function in piecewise without the absolute values.

(6 points)

$$f(x) = 2|7+x| - 3|x-4|$$

Q.7:

Given $f(x) = \frac{x}{x^2+1}$ and $g(x) = \frac{1}{x}$ find:

(8 points)

a) $(f+g)(x)$

b) $(f \circ g)(x)$ and state its domain

c) $(f \circ g)(1)$

d) $(g(f(2)))$

Q.8: Find the limits:

(21 points)

a) $\lim_{x \rightarrow \infty} \frac{5x^3 + 2x}{7x^4 + 3x^2 + 2}$

b) $\lim_{x \rightarrow -\infty} \frac{4x - 3}{\sqrt{7x^2 + 2}}$

c) $\lim_{x \rightarrow 2^-} \frac{x}{x^2 - 4}$

d) $\lim_{x \rightarrow 4} \frac{x - 4}{\sqrt{x + 5} - 3}$

e) $\lim_{x \rightarrow 0} \frac{\sin x - 7x}{x \cos x}$

f) $\lim_{x \rightarrow 0} \frac{2x + \sin 3x}{x}$

g) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

Q.9:

a) Find the value of c which makes the function continuous.

(8 points)

$$f(x) = \begin{cases} cx + 5 & x \leq 2 \\ 7x - 2 & x > 2 \end{cases}$$

b) Discuss the continuity of the function f . Find all the numbers, if any, at which f is not continuous.

$$f(x) = \begin{cases} 2x - 4 & x \leq -1 \\ 2x^2 + 6x - 4 & -1 < x \leq 1 \\ 4 & x > 1 \end{cases}$$

Q.10:

Suppose that f and g are continuous functions.

(5 points)

Given that $\lim_{x \rightarrow 4} f(x) = 0$, and $\lim_{x \rightarrow 4} g(x) = -3$.

Find: $\lim_{x \rightarrow 4} \frac{xg(x)}{\sqrt{f(x)} + 1}$

Q.11:

Use the definition of the limit to prove the following limit.

(5 points)

$$\lim_{x \rightarrow 16} \sqrt{x} - 1 = 3$$

Q.12:

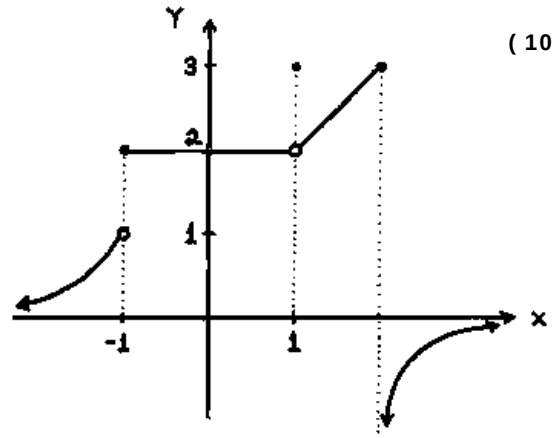
Sketch one period of the the graph of:

(5 points)

$$y = -5\sin(4x - \Pi)$$

(10 points)

Q.13: Consider the following graph,
Then answer the following questions:



a) $\lim_{x \rightarrow -1^+} f(x)$

b) $\lim_{x \rightarrow -1^-} f(x)$

c) $\lim_{x \rightarrow -1} f(x)$

d) $\lim_{x \rightarrow 2^+} f(x)$

e) $\lim_{x \rightarrow 2^-} f(x)$

f) $\lim_{x \rightarrow 2} f(x)$

g) $\lim_{x \rightarrow -\infty} f(x)$

h) $\lim_{x \rightarrow +\infty} f(x)$

i) Is $f(x)$ continuous at $x = 1$? Why or why not ?

Q.14: Use the graph of $y = x^2$ to sketch the graph of $y = -2(x+1)^2 + 3$ (5 points)