



Prince Sultan University

Department of Mathematical Sciences

Semester I, 2011 FALL (Term 111)
December 17, 2011

MATH 111 – Calculus I

Major II Exam

Time Allowed : 90 minutes

Maximum Points : 80 points

Name of the student: _____

ID number : _____

Section : _____

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 6 problems, some with several parts and a total of 5 pages. Make sure your paper has all these problems.

Question	Maximum score	Your Score
Q.1	26	
Q.2 , Q.3 , Q.4	24	
Q.5	18	
Q.6	12	
Total	80	

Q.1 (26 points): Find the derivative **(Simplify as much as possible)**

(i) $y = \left(\frac{8x - x^6}{x^3} \right)^{\frac{6}{5}}$

(ii) $y = \sin^5(x^3 + 2x^2)$

(iii) $y = (x^2)^{4x}$

(iv) $f(x) = \sqrt{\cos^{-1}(x^2)}$

(v) $y = \sqrt{4x + \cosh^2(5x)}$

(vi) $f(x) = \cos x \ln(1 + xe^{-x})$

Q.2 (8 points): Find the **equation(s)** of the tangent lines to the graph of $x^2 + 2xy - y^2 + x = 2$ at $x = 1$

Q.3 (6 points): Given that $x^2 - xy^2 = 1$. Find $\frac{d^2y}{dx^2}$ (in simplest form)

Q.4 (10 points): (i) Prove that the function $f(x) = x^3 + 3x + 15$ has exactly one real zero and it exists in the interval $[-3, 3]$.

(ii) Prove that $\left| \tan^{-1} x \right| < |x|$ for all $x \neq 0$. (Use Mean Value Theorem)

Q.5 (18 points): Evaluate the limits:

(i)
$$\lim_{x \rightarrow \infty} \frac{\ln(x^4 + x)}{\ln(x^3 + 2)}$$

(ii)
$$\lim_{x \rightarrow 0} \frac{\sin(\tan x)}{\sin x}$$

(iii)
$$\lim_{x \rightarrow \infty} \left(1 - \frac{2}{x} \right)^x$$

Q.6 (12 points): Consider the function $f(x) = x^3 - 3x + 1$. Find
(i) the critical numbers for f .

(ii) the absolute extrema for f on the interval $[-3, 2]$.

(iii) the intervals where f is increasing or decreasing.