Prince Sultan University

Department of Mathematical Sciences

Major III Exam

Semester I, 2005/2006 Fall (051)

24th December, 2005

MATH 111 – CALCULUS I

Time Allowed : 100 minutes **Mr. Khaled Naseralla**

Maximum Points: 100 points

Name of the student : _____

ID number : _____

Section : _____

For All The Students:

- **Answer all the questions.**
- **This exam consists of *a total of 7 pages and 11 questions.***
- **Show your working in the space provided for each question.**
- **Show all the key steps of your work.**
- **Scientific, non-programmable calculators are allowed.**

Question	Maximum score	*Your Score*
Q.1	16	
Q.2	8	
Q.3	8	
Q.4	8	
Q.5	10	
Q.6	10	
Q.7	8	
Q.8	9	
Q.9	9	
Q.10	7	
Q.11	7	
Total	**100**	

Q.1: **Find $\frac{dy}{dx}$ for the following functions:** **(16 points)**

a) $y = (3x^2 + 5)^{\frac{1}{x}}$

b) $y = \left(\frac{2x - 1}{e^x}\right)^2$

c) $y = x^3 (\sin x)^{\cos x}$

d) $y = (\tan x)^{\tan^{-1} x}$

e) $y = \left(\cos^{-1} x^{3}\right)^{4}$

f) $y = \dfrac{e^{x}\sqrt{x^{5}+2}}{(x+1)^{4}(x^{2}+3)^{2}}$

g) $y = x^{2}(\sin^{-1} x)^{3}$

h) $y = \dfrac{1}{e^{4x}+\ln 3x}$

Q.2: **Find the local linear approximation of** $f(x) = \dfrac{1}{\sqrt{1-x}}$ **at** $x_{\circ} = 0$ **(8 points)**

Q.3: **Let** $y = \dfrac{x}{x^2+1}$**. Find** dy **and** Δy **when** x **changes from** 2 **to** 2.5 **(8 points)**

Q.4: **Sketch a continuous curve of** $y = f(x)$ **with the stated properties:** **(8 points)**

a) $f(2) = 4$ **,** $f'(2) = 0$ **,** $f''(x) < 0$ **for all** x

b) $f(2) = 4$ **,** $f'(2) = 0$ **,** $f''(x) < 0$ **for** $x < 2$ **and** $f''(x) > 0$ **for** $x > 2$

Q.5: **Consider the function** $f(x) = x^6 - 6x^4$ **(10 points)**

a) Find the x **and** $y-$**values of all the critical points and identify each as a relative maximum, a relative minimum, or neither.**

b) Find the $x-$ **values of all the inflection points.**

Q.6: **Given** $f(x) = 3x^5 - 20x^3 + 10$ **, Find** **(10 points)**

a) **the intervals on which** f **is increasing.**
b) **the intervals on which** f **is decreasing.**
c) **the intervals on which** f **is concave up.**
d) **the intervals on which** f **is concave down.**
e) **The x-coordinates of the inflection points.**

Q.7: **For** $f(x) = \dfrac{x^3}{x^2 - 1}$ **,** **(8 points)**

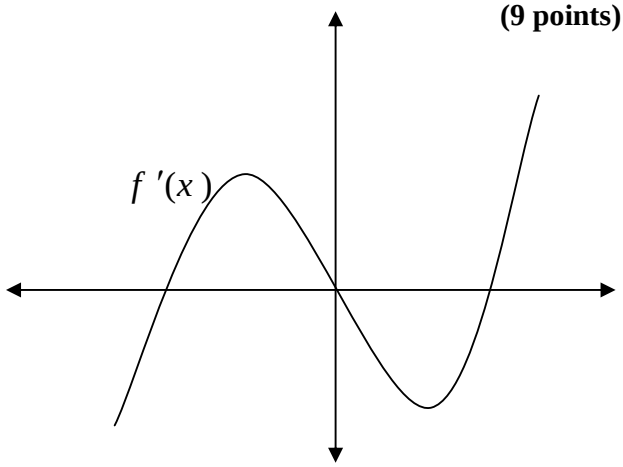
a) **Find the x-coordinates of all the critical numbers and classify them as stationary or non differentiable.**
b) **Find the relative extrema of the function and give their values.**

Q.8: **Graph the following polynomial. Label the x and y-intercepts, the critical points, and the inflection points.** **(9 points)**

$$f(x) = x^4 - 2x^2 + 1$$

Q.9:

Use the given graph of $f'(x)$ NOT $f(x)$ to find: **(9 points)**



a) The $x-$coordinates of the stationary points.

b) The intervals where f is increasing.

c) The intervals where f is decreasing.

d) The $x-$coordinates of the relative maxima.

e) The $x-$coordinates of the relative minima.

f) The intervals where f' is increasing.

g) The intervals where f' is decreasing.

h) The intervals where f is concave up.

i) The intervals where f is concave down.

Q.10: **At** $2:00 p.m.$ **two cars start moving from the same point.** **(7 points)**
One travels south at $60\ mi/hr$ **and the other travels west at** $25\ mi/hr$ **.**
At what rate is the distance between the two cars increasing at $5:00 p.m.$ **?**

Q.11: **A spherical balloon is blown up at a rate of** $5\ in^3/\sec.$ **At what rate is the radius of the balloon increasing when the radius is** $4\ inches$ **?** **(7 points)**