

Prince Sultan University Department of Mathematics and Physical Sciences

> Math 221 Second Midterm Exam Semester II, Term 132 Sunday, May 4, 2014

Time Allowed: 80 minutes

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Name: Student Number:

## **Statement of Ethics:**

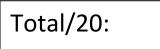
I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature:

A deal: You have to

Total/45:

- 1. show details of your work,
- 2. use the calculator upon proper calculations.



**Problem.1** (8 points) Derive a numerical differentiation formula of order  $h^4$  by applying Richardson extrapolation to

$$f'(x) = \frac{f(x+h) - f(x-h)}{h} - \frac{h^2}{6}f''' - \frac{h^4}{120}f^{(5)} - \dots$$

**Problem.2** (14 points) Consider the integral  $\int_{0}^{\pi} x^{2} \cos x dx$ 

- a) Evaluate the integral using analytical method.
- b) Use the Trapezoidal rule to approximate the integral.
- c) Use the Simpson's rule to approximate the integral. Which method is better?
- d) Find a bound of the error of the Trapezoidal rule.

**Problem.3** (12 points) Consider the system  $\begin{cases} 3x - y + z = 1\\ 3x + 6y + 2z = 0\\ 3x + 3y + 7z = 4 \end{cases}$ 

- a) Is the coefficient matrix diagonally dominant?
- b) Find the first 4 iterations of the Jacobi method of the system with  $X^{(0)} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ .
- c) Find the first 4 iterations of the Gauss Siedel method of the system with  $X^{(0)} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ .

**Problem.4** (11 points) Determine constants a, b, c and d that will produce a quadrature formula

$$\int_{-1}^{1} f(x)dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

That has degree of precision 3.