Prince Sultan University

Department of Mathematics and Physical Sciences

Math 221
Second Midterm Exam
Semester II, Term 132
Sunday, May 4, 2014

Time Allowed: 80 minutes

Name:
Student Number:

Statement of Ethics:

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature:

Total/45:

A deal: You have to

1. show details of your work,
2. use the calculator upon proper calculations.

Total/20:

Problem.1 (8 points) Derive a numerical differentiation formula of order h^4 by applying Richardson extrapolation to

$$f'(x) = \frac{f(x+h) - f(x-h)}{h} - \frac{h^2}{6} f''' - \frac{h^4}{120} f^{(5)} - \ldots.$$

Problem.2 (14 points) Consider the integral $\int_0^{\pi} x^2 \cos x dx$

a) Evaluate the integral using analytical method**.**
b) Use the Trapezoidal rule to approximate the integral.
c) Use the Simpson's rule to approximate the integral. Which method is better?
d) Find a bound of the error of the Trapezoidal rule.

Problem.3 (12 points) Consider the system $\begin{cases} 3x - y + z = 1 \\ 3x + 6y + 2z = 0 \\ 3x + 3y + 7z = 4 \end{cases}$.

a) Is the coefficient matrix diagonally dominant?

b) Find the first 4 iterations of the Jacobi method of the system with $X^{(0)} = [0 \quad 0 \quad 0]^T$.

c) Find the first 4 iterations of the Gauss Siedel method of the system with $X^{(0)} = [0 \quad 0 \quad 0]^T$.

Problem.4 (11 points) Determine constants a, b, c and d that will produce a quadrature formula

$$\int_{-1}^{1} f(x)dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

That has degree of precision 3.