

Prince Sultan University
Department of Mathematical Sciences
Major I Exam

Semester II, 2012 SPRING (112)
17th MARCH 2012

MATH 111 – CALCULUS I

Time Allowed : 90 minutes

Maximum Points: 100 points

Name of the student: _____

ID number : _____

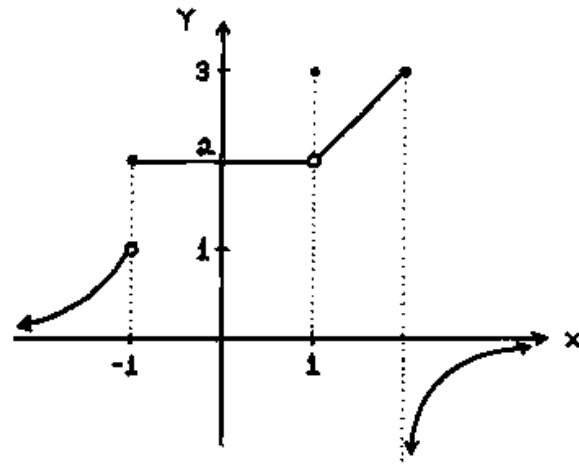
Section : 221(Mr. Khaled) or 222(Dr. Hamdi)

For All The Students:

- Answer all the questions.
- This exam consists of **a total of 7 pages and 12 questions.**
- Show your working in the space provided for each question.
- Show all the key steps of your work.
- Scientific, non-programmable calculators are allowed.

Question	Maximum score	Your Score
1 , 2 , 3	17	
4 , 5	14	
6 , 7 , 8	20	
9 , 10	33	
11 , 12	16	
Total	100	

Q.1 (7 points): Use the following graph to answer the following



a) $\lim_{x \rightarrow -1^+} f(x)$

b) $\lim_{x \rightarrow -1^-} f(x)$

c) $\lim_{x \rightarrow -1} f(x)$

d) $\lim_{x \rightarrow 2^+} f(x)$

e) $\lim_{x \rightarrow 2^-} f(x)$

f) $\lim_{x \rightarrow -\infty} f(x)$

g) $f(1)$

Q.2 (4 points): Use the Intermediate Value Theorem to show that the equation $4x^3 - 6x^2 + 3x - 2 = 0$ has a solution between 1 and 2.

Q.3 (6 points): If $h(x) = \frac{1+f(x)}{g(x)}$, $g(2)=1$, $g'(2)=2$, $f'(2)=5$, and $h'(2)=14$. Find $f(2)$

Q.4 (6 points): Find the derivative.

(i) $f(x) = \frac{x^2 - 2}{x^2 + 5x}$

(ii) $f(x) = (\sqrt{x} + 3x) \left(5x^2 - \frac{3}{x} \right)$

Q.5 (8 points): Find the values of a and b such that the function:

$$f(x) = \begin{cases} x^2 - a & \text{if } x < 1 \\ a + bx & \text{if } 1 \leq x \leq 2 \\ b - x^3 & \text{if } x > 2 \end{cases} \quad \text{is continuous everywhere}$$

Q.6 (7 points): Use the limit definition to find the derivative of $f(x) = 1 + \frac{1}{x}$ at $x = 1$

Q.7 (6 points): Find the horizontal asymptote(s) of the graph of the function $f(x) = \tan^{-1}\left(\frac{\sqrt{9x^2 + 2}}{3x + 7}\right)$

Q.8 (7 points): Find the point(s) on the graph of $f(x) = \frac{x-2}{x^3}$ at which the tangent line to the graph of f is parallel to the x -axis.

Q.9 (28 points): Evaluate the limits, if it exists.

a) $\lim_{x \rightarrow -\infty} \frac{x+3}{\sqrt{9x^2-5x}}$

b) $\lim_{x \rightarrow \infty} \left(\frac{4x}{x-1} \right) \left(\frac{x^2}{2+x^2} \right)$

c) $\lim_{x \rightarrow 5} \frac{3x}{(x-5)^5}$

d) $\lim_{x \rightarrow 2} \frac{x^4-16}{x^2-x-2}$

e) $\lim_{x \rightarrow 0} \frac{4x^2 - 5\sin^2 2x}{x^2}$

f) $\lim_{x \rightarrow 0^+} e^{-\frac{2}{x^3}}$

g) $\lim_{x \rightarrow 7} \frac{x-7}{\sqrt{7x}-7}$

Q.10 (5 points): If $x^3 - x + 4 \leq x + f(x) \leq 3x^2 + 1$ for all real numbers x , then find $\lim_{x \rightarrow 1} f(x)$.
Show your steps.

Q.11 (8 points): Consider the function $f(x) = |x^2 - 1|$. Discuss the differentiability of f .
Determine the value(s) of x where f is not differentiable (if any)

Q.12 (8 points): Consider the function $f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ e^x & \text{if } 0 \leq x \leq 1 \\ e - \ln x & \text{if } x > 1 \end{cases}$

Discuss the continuity of f . Determine the value(s) of x where f is not continuous (if any)