



Prince Sultan University
Department of Mathematics and Physical Sciences

Math 221
First Midterm Exam
Semester II, Term 132
Thursday, March 18, 2014

Time Allowed: 80 minutes

Name:
Student Number:

Statement of Ethics:

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature:

Total/50:

A deal: You have to

1. show details of your work,
2. use the calculator upon proper calculations.

Total/20:

Problem.1 (6 points) Find a bound for the number of iterations needed to achieve an approximation with accuracy 10^{-3} to the solution of $x^3 + x - 4 = 0$ lying in the interval $[1,4]$.

Problem.2 (8 points) Consider $p(z) = 3z^5 - 7z^4 - 5z^3 + z^2 - 8z + 2$.

a) Find a disk centered at the origin that contains all the zeros of $p(z)$.

b) Find a disk centered at the origin that contains none of the zeros of $p(z)$.

Problem.3 (12 points) Consider the following data in the table

x	-2	0	1
$f(x)$	0	1	-1

interpolates

- a) Use method of undetermined coefficients to find a polynomial that the data in the table.
- b) Find the Lagrange form of the interpolating polynomial for these data
- c) Write the Lagrange polynomial in the form $a + bx + cx^2$ and compare with the polynomial you get in part (a).

Problem.4 (10 points)

- a) Find the positive root of 85 using the following iteration formula $x_{n+1} = \frac{f(x_n)x_{n-1} - f(x_{n-1})x_n}{f(x_n) - f(x_{n-1})}$ by setting initially $x_0 = 7$ and $x_1 = 8$. Proceed until you reach x_4 and make your calculation up to four significant digits.
- b) Find bounds for the relative and absolute errors.

Problem.5 (12 points) Given $x^3 - x - 1 = 0$ on $[1,2]$ with $x_0 = 1$.

- a) Find F that is contractive on $[1,2]$ with $x_0 = 1$.
- b) Construct fixed-point iterations to determine a solution for $x^3 - x - 1 = 0$ on $[1,2]$ with $x_0 = 1$.