MATH 221 - Dr. Muhammad Islam Mustafa **Major Exam 2**

Student Name: ________________________________ **Time allowed: 60 minutes**

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- **Using the following data**

x	0	0.2	0.4	0.6
$f(x)$	0	0.20271	0.422793	0.684137

solve the questions 1-3.

<u>Q1.</u> Use appropriate Lagrange interpolating polynomial of degree two to approximate $f(0.5)$.

<u>Solution:</u>

<u>Q2.</u> Use the Newton forward or backward divided difference method, as suitable, to construct interpolating polynomial of degree three for the above data and approximate $f(0.5)$ using this polynomial

<u>Solution:</u>

x	$f(x)$	1^{st} divided difference	2^{nd} divided difference	3^{rd} divided difference
0	0			
0.2	0.20271			
0.4	0.422793			
0.6	0.684137			

Q3. Use the most accurate three-point formula to approximate $f'(0.2)$.

Solution:

Q4. Approximate the integral $\int_0^{0.5} e^{x^2}\,dx$ using the Trapezoidal rule.

Solution:

Q5. Determine the values of n and h required to approximate the integral $\int_1^{1.5} \ln x\,dx$ to within 10^{-5} using the Composite Simpson's rule. Compute the approximation and find the actual error.

Solution:

A list of formulas

The three-point formulas

$$f'(x_0) = \frac{1}{2h}[-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)] + \frac{h^2}{3} f'''(\xi)$$

$$f'(x_0) = \frac{1}{2h}[f(x_0 + h) - f(x_0 - h)] - \frac{h^2}{6} f'''(\xi)$$

The Midpoint rule

$$\int_a^b f(x)dx = 2hf(x_0) + \frac{h^3}{3} f''(\xi)$$

Trapezoidal rule

$$\int_a^b f(x)dx = \frac{h}{2}[f(a) + f(b)] - \frac{h^3}{12} f''(\xi)$$

Simpson's rule

$$\int_a^b f(x)dx = \frac{h}{3}[f(a) + 4f(x_0) + f(b)] - \frac{h^5}{90} f^{(4)}(\xi)$$

Composite Trapezoidal rule

$$\int_a^b f(x)dx = \frac{h}{2}[f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)] - \frac{b-a}{12} h^2 f''(\xi)$$

Composite Simpson's rule

$$\int_a^b f(x)dx = \frac{h}{3}[f(a) + 2\sum_{i=1}^{\frac{n}{2}-1} f(x_{2i}) + 4\sum_{i=1}^{\frac{n}{2}} f(x_{2i-1}) + f(b)] - \frac{b-a}{180} h^4 f^{(4)}(\xi)$$