

PRINCE SULTAN UNIVERSITY

MATH 111

Calculus

MAJOR EXAM III

Start : **4:00 pm**

End: **5:30 pm**

Name _____

I.D. _____

Section: **(8 AM)** **(10 A.M)** **(11 AM)** **(12 PM)**

1. Answer all questions
2. This exam consists of 5 pages, 9 questions
3. You can use a calculator, NOT a mobile phone.
4. No talking during the test.
5. Show all working out in the space provided.

Question No.	Max. Points	Points Scored
1	15	
2, 3, 4, 5	14	
6, 7,	12	
8, 9	9	
TOTAL	50	

Q1. (15 points) Find the derivative of the functions (Simplify your answers if possible)

a. $y = x^3 \ln x$

b. $y = \ln(\cosh x)$

c. $y = \ln \sqrt{\frac{x^2-5}{x+3}}$

d. $y = (\cos x)^x$

e. $y = \sinh^{-1}(\tan x)$

Q2. (3 points) Find equations of the tangent line to the curve $y = \ln(x^2 - 3x + 1)$ at the point where $x = 3$.

Q3. (4 points) The radius of a sphere is increasing at a rate of 4 mm/s. How fast is the volume of the sphere increasing when the diameter is 80 mm?

Q4. (3 points) Show that $\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$ for $-1 < x < 1$

Q5. (4 points) Find the absolute maximum and absolute minimum values of $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval $[-2, 3]$.

Q6. (5 points) Verify that the function $f(x) = e^{-2x}$ satisfies Mean Value Theorem on the interval $[0, 3]$. Then find all numbers c that satisfy the theorem.

Q7. (7 points) Given the function $f(x) = x^4 - 2x^2 + 3$.

- Find the intervals of increasing and decreasing
- Find the local minimum and maximum points
- Study the concavity and find the inflection points.

Q8. (4 points) Show that the equation $x^3 + e^{2x} = 0$ has exactly one real root.

Q9. (5 points) Sketch a graph of a function $f(x)$ that satisfies the following conditions:

- $f'(x) > 0$ if $-2 < x < 2$ and $f'(x) < 0$ if $|x| > 2$
- $f''(x) < 0$ if $0 < x < 3$ or $x < -3$ and $f''(x) > 0$ if $x > 3$ or $-3 < x < 0$,
- $\lim_{x \rightarrow \infty} f(x) = 1$ and $\lim_{x \rightarrow -\infty} f(x) = -1$
- $f(0) = 0, f(-2) = -4, f(2) = 4$