PRINCE SULTAN UNIVERSITY

MATH 111 CALCULUS

MAJOR EXAM 3 6th JUNE 2009

Start: 4:00 p.m.
End: 5:30 p.m.

Name: _____

I.D. _____

1. Answer all questions
2. This exam consists of 1 Cover Sheet & 4 Question Sheets with 11 questions.
3. You can use a calculator, **NOT** a mobile phone.
4. Show all working out in the space provided.

Question No.	Max. Points	Points Scored
1,2,3	18	
4,5,6	18	
7,8,9	26	
10,11	18	
TOTAL	**80**	
		%

1) [6 points] Given $x^2y + 3xy^3 - x = 3$, show using implicit differentiation that $\frac{dy}{dx} = \frac{1 - 2xy - 3y^3}{x^2 + 9xy^2}$.

2) [6 points] Given that $y = \frac{\sqrt[3]{x^2 + 3}\,\sqrt{x^3 - 1}}{4x^2 - x}$, find $\frac{dy}{dx}$.

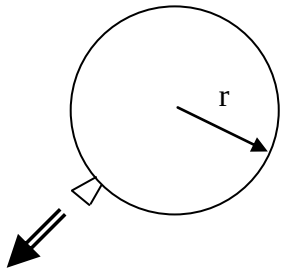
3) [6 points] Given that, find $\frac{dy}{dx}$ for the following:

a) $y = e^{x \tan x}$

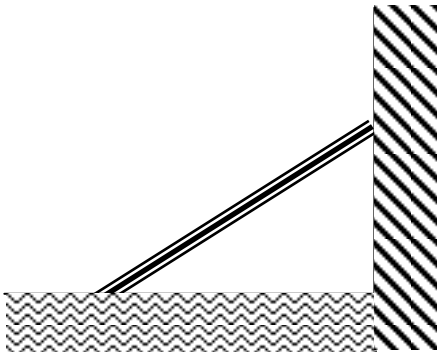
b) $y = e^{\left(x - e^{3x}\right)}$

4) [6 points] Given that $y = \left(x^2 - 3x\right)^{\ln x}$, use logarithmic differentiation to find $\frac{dy}{dx}$.

5) [6 points] A piece of spherical coal is burning such that its radius **decreases** at a constant rate of 15mm/min. At what rate is the volume reducing when the radius is 9 mm? *Hint: The volume of a sphere is given by* $V = \frac{4}{3}\pi r^3$.



6) [6 points] A 13-ft ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft/s, how fast will the bottom of the ladder be moving away from the wall when the top is 5 ft above the ground?



7) [6 points] Sketch a continuous curve that has **all** the following properties:

$f(-3)=12$
$f(0)=5$
$f(3)=0$

$f'(x)>0$ for $x>3$ and $x<-3$
$f'(-3)=f'(3)=0$

$f''(x)<0$ for $x<0$
$f''(x)>0$ for $x>0$

8) [10 points] Use the given graph of $f(x)$ to find:

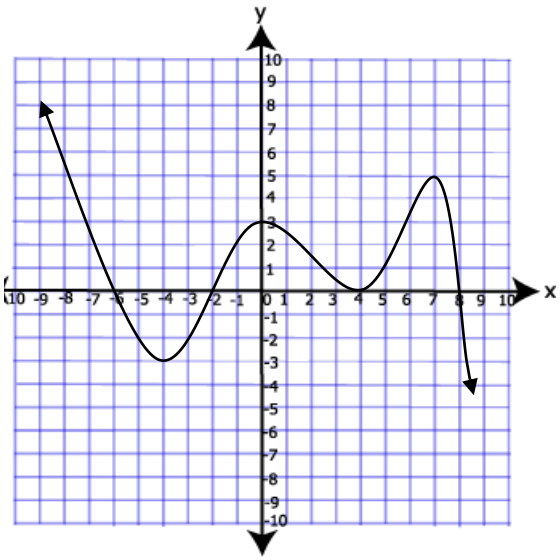
a) The increasing intervals

b) The decreasing intervals

c) The critical numbers

d) The relative maxima and minima

e) How many inflection points are there?



9) [10 points] Sketch the graph of the function $f(x)=x^3-3x^2+1$.
Label the x and y intercepts , critical points, and inflection points.

10) [6 points] Find $\frac{dy}{dx}$

a) $y = x^2(\sin^{-1} x)^3$

b) $y = \cot^{-1}\sqrt{1-x^2}$

11) [12 points] Given that $f(x) = x^4 - 18x^2 + 18$

a) Find the intervals on which f is increasing.

b) Find the intervals on which f is decreasing.

c) Find all the *x*-coordinates of the critical points.

d) Find all relative maxima and all relative minima.

e) Find all inflection points.

f) Find the open intervals on which f is concave up

g) Find the open intervals on which f is concave down.