

". You can use a calculator, NOT a mobile phone.

٤. No talking during the test.

•. Show all working out in the space provided.

Question No.	Max. Points	Points Scored
1,2	14	
3	14	
4	16	
5,6	18	
7,8,9	18	
10,11,12	20	
TOTAL	100	

1) [6 points] Determine whether or not the following maximum problems are in are in standard form, and explain why. **Do not attempt to solve them.** 

a) Maximize  $P = 3x_1 + 4x_2$  subject to the constraints  $x_1 + 4x_2 \le 74$  $x_1 \ge 0$   $x_2 \ge 0$ 

b) Maximize  $P = 2x_1 + x_2$  subject to the constraints  $-x_1 + x_2 - 3x_3 \ge 8$  $x_1 \ge 0$   $x_2 \ge 0$   $x_3 \ge 0$ 

7) [8 points] The following maximum problem is **not** in standard form.

Maximize  $P = 2x_1 + 4x_2 + x_3$  subject to the constraints  $\begin{array}{l}
-2x_1 - 3x_2 + x_3 \ge -8 \\
-3x_1 + x_2 - 2x_3 \ge -12 \\
2x_1 + x_2 + x_3 \le 10 \\
x_1 \ge 0 \quad x_2 \ge 0 \quad x_3 \ge 0
\end{array}$ 

a) Re-write the problem so that it is in standard form.

b) Introduce Slack Variables and re-write the Objective Function equal to zero.

c) Set up the initial Simplex Tableau. Do not attempt to solve it.

3) [14 points] Use the Simplex Method to solve the following problem:

 $x_1 + 3x_2 + 2x_3 \le 30$ Maximize  $P = 4x_1 + 2x_2 + 5x_3$  subject to the constraints  $2x_1 + x_2 + 3x_3 \le 12$  $x_1 \ge 0$   $x_2 \ge 0$   $x_3 \ge 0$  4) [16 points] Use the Mixed Constraints method to solve the following problem:

Minimize 
$$z = 3x_1 + 6x_2 + 2x_3$$
 subject to the constraints  

$$\begin{aligned}
x_2 + x_3 &\leq 4 \\
x_1 + 4x_2 + x_3 &\leq 3 \\
x_1 + x_2 + x_3 &\geq 1 \\
x_1 &\geq 0 \qquad x_2 &\geq 0 \qquad x_3 &\geq 0
\end{aligned}$$

•) [8 points] If U = universal set = {0,1,2,3,4,5,6,7} and if  $A = \{3,5,6,7\}$ ,  $B = \{1,2,3,4,5\}$  and  $C = \{2,3,4,5\}$  find a)  $A \cap B$ b)  $(\overline{A} \cup B) \cap C$ 

c)  $A \cup (B \cap C)$ 

d)  $\overline{A} \cap \overline{C}$ 

6) [10 points] A survey of PSU students showed the following information about the sports they play.

29 played football
26 played tennis
25 played basketball
13 played football and tennis
11 played tennis and basketball
18 played football and basketball
6 played all three sports
7 played none of them

Part a) Draw a Venn Diagram to answer the following questions:

Part b

i) How many of these students played only tennis? .....

ii) How many played tennis and basketball but not football? .....

iii) How many played neither football nor tennis?.....

iv) How many played football or tennis or both? .....

v) How many students were surveyed? .....

- 7) [6 points] Calculate the following:
- a) The number of ways of choosing 5 students from a group of 10 and arranging them in a line.
- b) The number of ways 5 different boxes can be arranged on top of each other?
- c) The number of ways of selecting seven books from a shelf containing twelve.
- A) [6 points] New British license plates consist of five letters and two numbers (assume all 26 letters and 10 digits are allowed). How many plates are possible:
- a) With no repeated numbers?
- b) With no repeated letters?
- c) With repeated letters and numbers allowed?
- <sup>9</sup>) [6 points] ] From 12 women and 10 men a committee of 5 is to be formed. The committee must include at least 1 woman and 1 man. In how many ways can this be done?

())[8 points] A shop display needs 2 green, 3 yellow, 2 blue and 5 white light bulbs for its window display. If there are 4 green, 8 yellow, 4 blue and 10 white bulbs available, how many different arrangements are possible if the order of the light bulbs does **not** matter?

11) [8 points] Expand  $(2x+2y)^5$  using the Binomial theorem.

17) [4 points] Show that 
$$\binom{8}{5} = \binom{4}{4} + \binom{5}{4} + \binom{6}{4} + \binom{7}{4}$$