

**PRINCE SULTAN UNIVERSITY****MATH 111****CALCULUS****MAJOR EXAM 1**    **1 April 2009****Start:**        **4:00 p.m.****End:**         **5:30 p.m.****Name:** \_\_\_\_\_**I.D.** \_\_\_\_\_**Instructor:** \_\_\_\_\_ **Section:** \_\_\_\_\_

1. Answer all questions
2. This exam consists of 1 Cover Sheet & 5 Question Sheets with 10 questions.
3. You can use a calculator, **NOT** a mobile phone.
4. No talking during the test.
5. Show all working out in the space provided.

Question No.	Max. Points	Points Scored
1,2	20	
3	16	
4	16	
5,6,7	16	
8,9,10	12	
TOTAL	80	

1) [12 points] Find the value of the following limits:

a)  $\lim_{x \rightarrow 0} \frac{\tan 8x}{\sin 4x}$

b)  $\lim_{t \rightarrow 0} \frac{t^2}{1 - \cos^2 5t}$

c)  $\lim_{\theta \rightarrow 0} \frac{\tan A\theta^2 + \sin^2 B\theta}{\theta^2}$

2) [8 points] Given that  $y = 2x^2 - 3x$ ,

a) Find the average rate of change over the interval  $[3, 4]$ .

b) Find the instantaneous rate of change of  $y$  with respect to  $x$  at an arbitrary value of  $x_0$  using the limit of the **Difference Quotient**.

3) [16 points] Find  $\frac{dy}{dx}$ . Simplify your answer as much as possible.

a)  $y = x^2 \sin^3(2x)$

b)  $y = x \cos(5x) - \sin^2 x$

c)  $y = (x + \csc(x^3 + 3))^{-3}$

d)  $y = \left( \frac{5x}{3x+2} \right)^3$

4) [16 points] Find  $f'(x)$ . Simplify your answer as much as possible.

a)  $f(x) = x^8 - 3\sqrt{x} + 5x^{-3}$

b)  $f(x) = \frac{4x^2 - x + 3}{\sqrt{x}}$

c)  $f(x) = \left(\frac{3x+2}{x}\right)(x^{-5} + 1)$

d)  $f(x) = (3x-5)^4(7-x)^{10}$

5) [4 points] Find  $\frac{d^2y}{dx^2}$  for  $y = x^5 \cos x$

6) [6 points] Show that  $y = x \sin x$  is a solution to  $y'' + y = 2 \cos x$

7) [6 points] Find the  $x$ -coordinates of all points at which the graph of  $f(x) = \frac{(x-2)}{x^3}$  has a horizontal tangent line.

- 8) [6 points] Find an equation for the tangent line to the graph of  $y = \tan(4x^2)$  at  $x = \sqrt{\pi}$

- 9) [6 points] Given that  $f(2) = 1$ ,  $f'(2) = 7$ , and  $g(2) = 1$ ,  $g'(2) = -5$

- a) Find  $g'(2)$ , where  $g(x) = [f(x)]^3$

- b) Find  $h'(2)$ , where  $h(x) = \frac{f(x)}{g(x)}$