Prince Sultan University

Department of Mathematical Sciences

Major II Exam

Semester I, 2012 FALL(121)

8th December 2012

MATH 111 – CALCULUS I

Time Allowed : 90 minutes
Maximum Points: 80 points

Name of the student: _____

ID number : _____

Dr. Abdelouahid Hamdi		Mr. Abid Zargar	Mr. Khaled Naseralla
Section 250	Section 224	Section 249	Section 223
10 ---- 11	11 ----- 12	8 ----- 9	10 ----- 11

For All The Students:

- You may use a **SCIENTIFIC CALCULATOR** that does **NOT** have **GRAPHING** capabilities.
- You may **NOT** borrow a calculator from anyone.
- Answer all the questions.
- This exam consists of *a total of 7 pages and 10 questions.*
- Show all the key steps of your work in the space provided for each question. Please indicate your **FINAL** answers clearly.
- There should be **NO** talking during the exam.

Question	Maximum Points	*Points Earned*
1 , 2	14	
3 , 4 , 5	17	
6 , 7	12	
8 , 9	13	
10	24	
Total	**80**	

Q.1 (6 points): If $y = e^x \sin(x)$. Show that $y'' - 2y' + 2y = 0$

Q.2 (8 points): Find all the points where the curve $4x^2 + y^2 - 8x + 4y + 4 = 0$ has;

(i) Horizontal tangent lines

(ii) Vertical tangent lines

Q.3 (4 points): Prove the identity: $\cosh(2x) = \cosh^2(x) + \sinh^2(x)$

Q.4 (8 points): Find the equations of the **tangent and normal lines** to the graph of the curve
$xy^3 - 3x^2 = 5$ at $x = 1$

Q.5 (5 points): A cylindrical water tank with a radius of $4m$. Water is being pumped from the tank at a rate of $2.4m^3/\min$. Find the rate at which the water level is decreasing.

Q.6 (6 points): Use the limit definition of the derivative to find $f'(4)$ for $f(x) = \sqrt{x-2}$

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Q.7 (6 points): Find the limit:

(i) $\lim_{x \to 0} \frac{\sin(4x)}{\sin(5x)\cos(x)}$

(ii) $\lim_{\theta \to 0} \frac{\sin\theta}{\theta + \tan\theta}$

Q.8 (5 points): Determine whether f is differentiable at $x = 1$ or not. Show your work

$$f(x) = \begin{cases} x^2 + 2 & \text{if } x \le 1 \\ x + 2 & \text{if } x > 1 \end{cases}$$

Q.9 (8 points): Find the derivative of each function

(i) $f(x) = \dfrac{1}{x + \sqrt{1 - x^2}}$

(ii) $k(x) = x^2 \cos(3x)$

Q.10 (24 points): Find the derivative of each function

(iii) $f(x) = e^{5x} \ln\left(\tanh(\frac{x}{2}) \right)$

(iv) $g(x) = \cosh^{-1}(5x)$

(v) $y = \dfrac{e^{x^2}\sqrt{x^2+3}}{(x^3+2)^5(x^2+1)^2}$

(vi) $y = \left(x^2 - 3x + 5 \right)^{\cos(x)}$

(vii) $g(x) = e^{\tan^{-1}\sqrt{3x}}$

(viii) $f\ (x) = \log_3(x)\left[\sin^{-1}(x) \right]^2$