Prince Sultan University
Orientation Mathematics Program
MATH 223
Class Major Test I
Semester II, Term 162

Time Allowed: **90** minutes

Q1. (4 Points) Let $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 2 & 5 \end{bmatrix}$.

Evaluate the following
  a. tr(AB)
  b. $(2A + B)^T$

Q2.   (4 Points) Show that if A is invertible, then AA^T is also invertible.

Q3.   (4 Points) Let $f(x) = x^2 - 2x$ be a function. Evaluate f(A) where $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

Q4. (8 Points) Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ be a matrix such that det (A) $= -2$.

Evaluate the following:
  a. $\det(3A) =$
  b. $\det(3A^{-1}) =$
  c. $\det((3A)^{-1}) =$

  d. $\det(B) =$, where $B = \begin{bmatrix} g & h & i \\ d + 2a & e + 2b & f + 2c \\ 5a & 5b & 5c \end{bmatrix}$.

Q5.   (4 Points) Use Cramer's rule to find the value of x only.
$$x - 4y + z = 6$$
$$4x - y + 2z = -1$$
$$2x + 2y - 3z = -2$$

Q6.   (5 Points) Show that for any vectors u, v in R^n, the following equality holds:
$$\|u + v\|^2 + \|u - v\|^2 = 2(\|u\|^2 + \|v\|^2)$$

Q7.   (4 Points) Find the distance between the point (1, –3, 1) and the plane $2x - 2y + z = 5$.

Q8.   (6 Points) Find the vector, parametric equations and the standard equation of the plane containing the point (2, –2, 5) and parallel to the vectors (3, 0, 3) and (– 4, 3, 1).

Q9.   (4 Points) Show that for any vectors u, v in R^3 and a scalar k we have
$$k(u \times v) = (ku) \times v$$

Q10. (3 Points) Find a vector w that is perpendicular to the vectors (–2, 3, 5) and (2, –1, 2).

Q11. (4 Points) Find the area of the parallelogram in R^2 determined by the vectors $u = (-2, 3)$ and $v = (1, 5)$.