



# PRINCE SULTAN UNIVERSITY

MATH 101

FINITE MATHS

MAJOR 2 EXAM

15<sup>TH</sup> DEC. 2012

Time Allowed: 90 minutes

Name: \_\_\_\_\_

I.D.: \_\_\_\_\_

Section or Time: \_\_\_\_\_

1. Answer all questions.
2. This exam consists of 5 pages including this cover sheet.
3. There are 9 questions. CHECK YOU HAVE ALL THE QUESTIONS.
4. You can use a calculator, NOT a mobile phone.
5. Show all working out in the space provided. If you use the back of a page indicate this on the front.

Question No.	Max. Points	Points Scored
1,2	20	
3,4	18	
5,6	18	
7,8,9	14	
TOTAL	70	
TOTAL	20	

1) [8 points] A factory manufactures two kinds of ice skates – racing skates and figure skates. The racing skates require 6 work-hours in the fabrication department, whereas the figure skates require 4 work-hours there. The racing skates require 1 work-hour in the finishing department, whereas the figure skates require 2 work-hours there. The fabricating department has at most 120 work-hours per day, and the finishing department has at most 40 work-hours per day available. Each racing skate brings a profit of \$10 and each figure skate a profit of \$12. Assuming that every skate made will be sold, use a graph of inequalities to find how many of each should be made to maximize the profit?

2) [12 points] Use the Duality Principle to solve the following problem:

$$x_1 + x_2 \geq 4$$

Minimize  $C = 6x_1 + 3x_2$  subject to the constraints  $3x_1 + 4x_2 \geq 12$

$$x_1 \geq 0, \quad x_2 \geq 0$$

3) [6 points] Determine which of the following problems are in standard form. If a problem is not in standard form consider whether the problem can be written in standard form or not and explain why.

$$\begin{aligned} & 3x_1 + x_2 \leq 10 \\ & 4x_1 - x_2 \leq 5 \\ \text{a) Maximize } z = 6x_1 - 8x_2 + x_3 \text{ subject to the constraints } & x_1 - x_2 - x_3 \geq -3 \\ & x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0 \end{aligned}$$

$$\begin{aligned} & 6x_1 + 8x_2 \geq 12 \\ \text{b) Maximize } z = 16x_1 + 2x_2 \text{ subject to the constraints } & 2x_1 + 3x_2 \leq 6 \\ & x_1 \geq 0, \quad x_2 \geq 0 \end{aligned}$$

$$\begin{aligned} & 2x_1 - 3x_2 + 3x_3 \leq 14 \\ \text{c) Minimize } C = x_1 + 2x_2 + 2x_3 \text{ subject to the constraints } & 2x_1 + 2x_2 + x_3 \geq 1 \\ & x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0 \end{aligned}$$

4) [12 points] Use the Mixed Constraints Method to solve the following problem:

$$\begin{aligned} & x_1 + x_2 \leq 12 \\ & 5x_1 + 2x_2 \geq 36 \\ \text{Maximize } P = 3x_1 + 4x_2 \text{ subject to the constraints } & 7x_1 + 4x_2 \geq 14 \\ & x_1 \geq 0 \quad x_2 \geq 0 \end{aligned}$$

5) [8 points] Analyse the following tableaus. For each tableau determine whether it indicates no solution or is the final tableau (if so, give the solution) or requires further pivoting (if so, circle the pivot element). **Write a brief sentence to explain your choice.**

a) 
$$\begin{array}{c|cccc|c} P & x_1 & x_2 & s_1 & s_2 & RHS \\ \hline 0 & 1 & 0 & 0 & 4 & 24 \\ 0 & 0 & -2 & 1 & 4 & 36 \\ \hline 1 & 0 & -10 & 0 & 4 & 20 \end{array}$$

b) 
$$\begin{array}{c|ccccc|c} P & x_1 & x_2 & s_1 & s_2 & RHS \\ \hline 0 & \frac{1}{2} & 0 & 1 & 1 & 20 \\ 0 & \frac{1}{3} & 1 & 0 & 1 & 30 \\ \hline 1 & -1 & 0 & 0 & 1 & 120 \end{array}$$

c) 
$$\begin{array}{c|ccccc|c} P & x_1 & x_2 & s_1 & s_2 & RHS \\ \hline 0 & -3 & 0 & 1 & 1 & -2 \\ 0 & 1 & -2 & 0 & 1 & -8 \\ \hline 1 & -1 & 0 & 0 & 1 & 120 \end{array}$$

d) 
$$\begin{array}{c|ccccc|c} P & x_1 & x_2 & s_1 & s_2 & RHS \\ \hline 0 & 2 & 0 & 1 & 1 & -6 \\ 0 & 2 & 1 & 0 & 1 & -4 \\ \hline 1 & -1 & 0 & 0 & 1 & 120 \end{array}$$

6) [10 points] If  $U = \{1, 2, 3, 4, 5, 6, 7\}$  and if  $A = \{1, 4, 5, 6\}$ ,  $B = \{1, 3, 6\}$  and  $C = \{3, 4, 5, 6, 7\}$  find:

a)  $A \cup \overline{B}$

b)  $\overline{A} \cap \overline{B}$

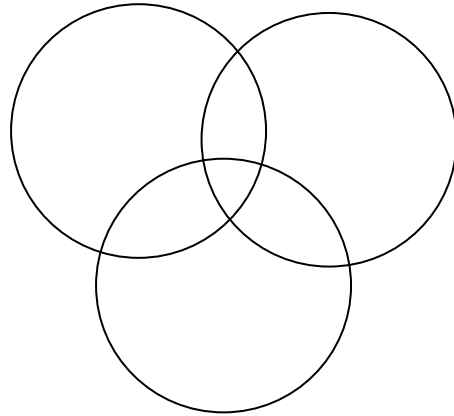
c)  $\overline{A \cup B}$

d)  $A \cup (B \cap C)$

e)  $\overline{A \cap C}$

7) [10 points] A survey of students showed the following information about their favorite TV shows. Draw a Venn Diagram.

48 watch documentaries  
38 watch movies  
29 watch comedies  
18 watch documentaries and movies  
19 watch movies and comedies  
16 watch documentaries and comedies  
8 watch all three types  
15 watch none of them



- i) How many watch movies but not documentaries nor comedies? .....
- ii) How many watch movies and comedies but not documentaries? .....
- iii) How many watch only documentaries? .....
- iv) How many watch neither movies nor documentaries? .....
- v) How many watch either comedies or documentaries or both, but not movies? .....
- vi) How many customers were surveyed? .....

8) [2 points] An e-mail password consists of 6 characters. The first 3 characters can be a letter from A to G (including A and G). The last 3 characters can be a digit from 0 through to 5 (including 0 and 5). How many passwords are possible if repetition is allowed?

9) [2 points] A telephone company issues phone numbers in the format 05ABCDEF. A and B can be the digit 3, 6 or 7. The customer can choose any digit for C,D,E and F. If repetition of digits is **NOT** allowed, how many phone numbers are possible?