



# Prince Sultan University

MATH 111

## Major Test II

Semester I, Term 151

Monday, November 23, 2015

Time Allowed: **90 minutes**

Student Name: \_\_\_\_\_

Student ID #: \_\_\_\_\_

Teacher's Name: \_\_\_\_\_ Class Time: \_\_\_\_\_

Section #:

Serial Class #:

### **Important Instructions:**

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 10 problems, some with several parts. Make sure your paper has all these problems.

| Problems     | Max points | Student's Points |
|--------------|------------|------------------|
|              |            |                  |
| 1            | 20         |                  |
| 2, 3,4       | 20         |                  |
| 5, 6, 7      | 18         |                  |
| 8, 9, 10     | 22         |                  |
|              |            |                  |
| <b>Total</b> | <b>80</b>  |                  |

Q1. (20 points) Find the derivative  $y'$  of the following function and simplify where necessary

(a)  $y \sin^2 x - \sin^2(xy) = \cos x$

(b)  $y = \tanh^2(\ln x) + \ln(\tan^{-1}(3x^5))$

(c)  $x^{2y} = (2y)^x$

(d)  $y = \ln\left(\frac{(x^2 + e^{x^3})(x - \ln x^2)}{(x^2 - \sin x)^4}\right)$

Q2. (8 Points) Suppose that  $g(x) = (f(x) + xe^{2x})^5$ , and that at  $x = 0$ ,  $f(0) = -1$ ,  $f'(0) = 0$ .

a. Evaluate the derivative  $g'(0)$

b. Find the equation of the **tangent and normal** to  $y = g(x)$  at  $x = 0$ .

Q3. (8 Points) Find the limit below using limit laws:

1.  $\lim_{x \rightarrow 0} \frac{\sin 2x \cdot \cos^2 x \cdot \tan 3x}{x \cdot \sin 5x}$

2.  $\lim_{x \rightarrow 1} \frac{\sin(2x - 2)}{x^2 - 1}$

Q4. (4 Points) Use definitions to prove the identity:

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

Q5. (5 Points) Use logarithmic differentiation to find  $y'$  where  $y = \frac{3x^{\cos x} (\sin x - 1)^3}{\sqrt{x^3 - 1}}$

Q6. (5 Points) Find the 20<sup>th</sup> derivative of:  $y = -\ln(1 + x)$ .

Q7. (8 Points) Find the critical numbers of the function

1.  $f(x) = x^5 e^{-3x}$

2.  $g(x) = x^{1/3} - x^{-2/3}$

Q8. (8 Points) At noon, ship A is 50 km west of ship B. Ship A is sailing south at 30 km/h and ship B is sailing north at 20 km/h. How fast is the distance between the ships changing at 3:00 PM?

Q9. (6 Points) A cylindrical tank with diameter 8 meters is being filled with water at a rate of 5 cubic meters per minute. How fast is the height of the water increasing?

Q10.(8 Points) Find the absolute maximum and absolute minimum values of

1.  $f(x) = 2x^3 - 3x^2 - 12x + 1$  on the interval  $[-4, 1]$ .

2.  $g(x) = \sqrt[3]{x}(8 - x)$  on the interval  $[0, 8]$ .