

PRINCE SULTAN UNIVERSITY

Department of Mathematical Sciences

MATH 002 Midterm Examination

Sunday, 26 March 2006

(052)

Time allowed: 90 minutes

Student Name: _____

Student ID number: _____

Section: _____

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. If your mobile phone is seen or heard, your exam will be taken immediately.
6. You must show all your work beside the problem. Be organized.
7. You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.
8. This examination has 15 problems, some with several parts. Make sure your paper has all these problems.

Problems	Max points	Student's Points
1, 2, 3, 4	22	
5, 6, 7	20	
8, 9, 10	22	
11, 10, 13	22	
14, 15	14	
Total	100	

1. (6 points) Approximate each expression using a calculator. Round your answer to three decimal places.

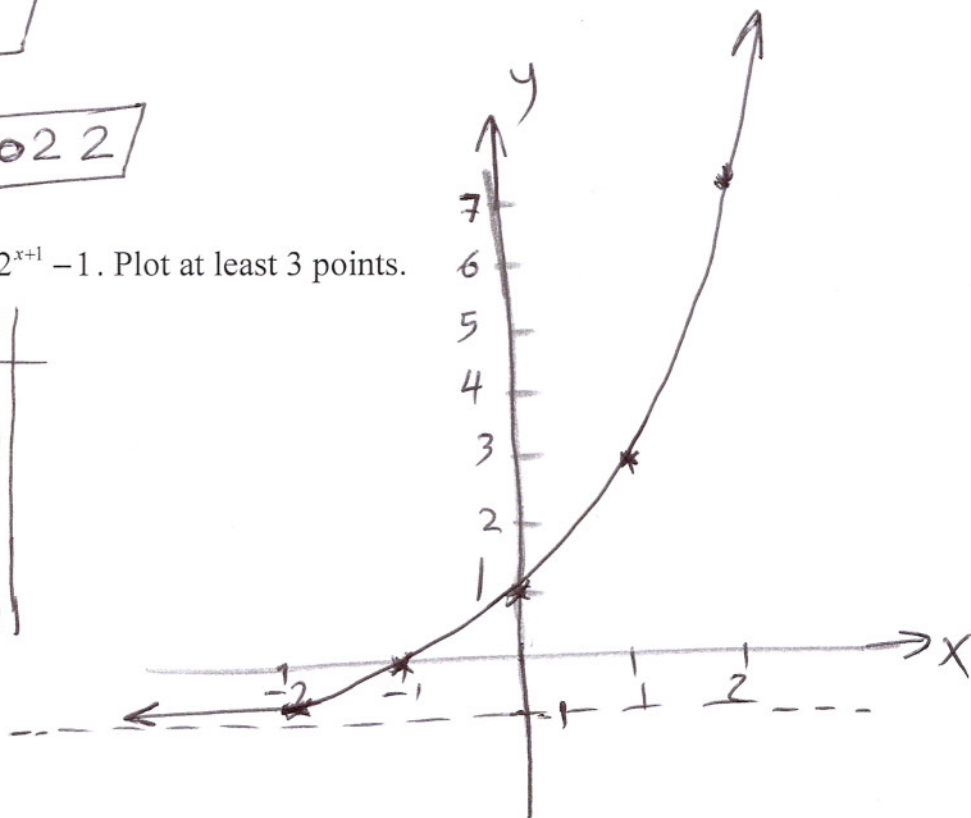
(a) $4^{\sqrt{0.5}} = \boxed{2.665}$

(b) $e^{-2.19} = \boxed{0.112}$

(c) $\log_7(2506) = \boxed{4.022}$

2. (6 points) Graph $f(x) = 2^{x+1} - 1$. Plot at least 3 points.

x	$f(x) = 2^{x+1} - 1$	(x, y)
-2	$-\frac{1}{2}$	$(-2, -\frac{1}{2})$
-1	0	$(-1, 0)$
0	1	$(0, 1)$
1	3	$(1, 3)$
2	7	$(2, 7)$



3. (6 points) Convert each angle in radians to degrees. Round to two decimal places.

(a) -4.8 radians = $\boxed{-275.02^\circ}$

(b) $\frac{\pi}{17}$ radians = $\boxed{10.59^\circ}$

4. (4 points) Find the measure of the central angle, in degrees of a circle of radius 6 centimeters that intercepts an arc of length 27 centimeters.

$$S = r\theta$$

$$\theta = \frac{S}{r} = \frac{27}{6} = 4.5 \text{ radians}$$

$$= 257.83^\circ$$

$$\approx \boxed{258^\circ}$$

5. (6 points) The annual amount that we spend to attend sporting events can be modeled by $f(x) = 2.05 + 1.3 \ln x$ where x represents the number of years after 1984 and $f(x)$ represents the total annual expenses for admission, in billions of dollars. In 2000, approximately how much was spent on admission?

$$t = 2000 - 1984 = 16$$

$$f(16) = 2.05 + 1.3 \ln(16)$$

$$f(16) = \boxed{5.65 \text{ billion dollars}}$$

6. (8 points) Solve $e^{4x-5} - 7 = 11243$. Use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.

$$e^{4x-5} = 11243 + 7$$

$$e^{4x-5} = 11250$$

$$\ln e^{4x-5} = \ln(11250)$$

$$4x - 5 = \ln(11250)$$

$$4x = \ln(11250) + 5$$

$$x = \frac{\ln(11250) + 5}{4} = \boxed{3.58}$$

7. (6 points) Write the logarithmic expression $\frac{1}{3}(\log_4 x - \log_4 y) + 2 \log_4(x+1)$ as a single logarithm whose coefficient is 1.

$$\frac{1}{3} \log_4 \frac{x}{y} + \log_4 (x+1)^2$$

$$= \boxed{\log_4 \sqrt[3]{\frac{x}{y}} \cdot (x+1)^2}$$

8. (6 points) Solve $3\log_2(x-1) = 5 - \log_2 4$.

$$\begin{aligned} \frac{3\log_2(x-1)}{3} &= \frac{5-2}{3} = \frac{3}{3} \\ \log_2(x-1) &= 1 \\ x-1 &= 2^1 \\ \boxed{x=3} \end{aligned} \quad \left\{ \begin{matrix} 3 \end{matrix} \right\}$$

9. (6 points) Use a calculator to find the value of the trigonometric function to four decimal places.

i. $\cot \frac{\pi}{18} = \boxed{5.6713}$

ii. $\csc 17^\circ = \boxed{3.4203}$

iii. $\cos^{-1} \frac{\sqrt{5}}{7} = \boxed{71.3710^\circ}$

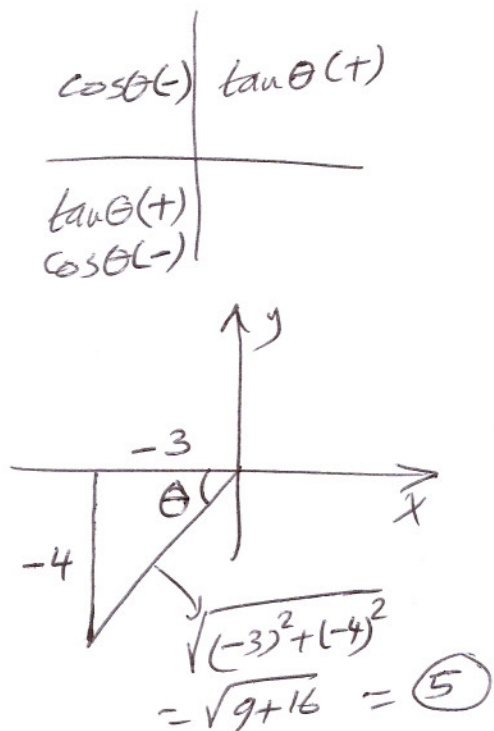
10. (10 points) Let $\tan \theta = \frac{4}{3}$ and $\cos \theta < 0$. Find the **exact** value of each of the remaining trigonometric functions of θ .

θ is Q.III

$$\sin \theta = -\frac{4}{5} \rightarrow \csc \theta = -\frac{5}{4}$$

$$\cos \theta = -\frac{3}{5} \rightarrow \sec \theta = -\frac{5}{3}$$

$$\tan \theta = \frac{4}{3} \rightarrow \cot \theta = \frac{3}{4}$$



11. (10 points) Determine the amplitude, period and phase shift of $y = -2\cos(2x - \pi)$. Then graph one period of the function. Show the coordinates of the five key points on the graph.

Amplitude, $A = |-2| = \boxed{2}$

period, $P = \frac{2\pi}{2} = \boxed{\pi}$

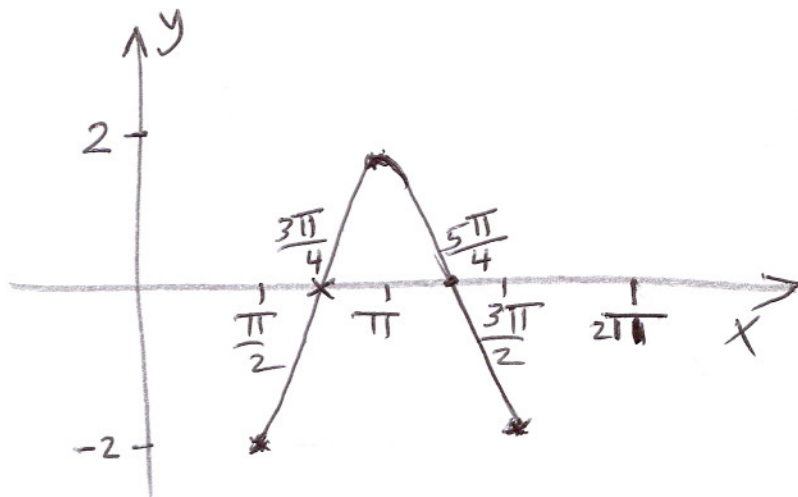
phase shift $\rightarrow 2x - \pi = 0$
 $2x = \pi$
 $x = \frac{\pi}{2}$

Key points

start $\rightarrow x = \frac{\pi}{2}$

End $\rightarrow x = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$

Middle $\rightarrow x = \frac{\frac{\pi}{2} + \frac{3\pi}{2}}{2} = \boxed{\pi}$



12. (6 points) Solve the right triangle shown below. Round length to two decimal places and express angles to the nearest tenth of a degree.

$B = 16.8^\circ$, $b = 30.5$

$\tan(16.8^\circ) = \frac{30.5}{a}$

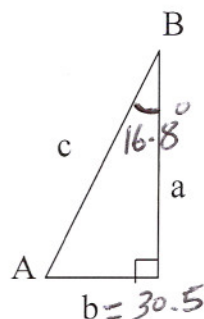
$a \cdot \tan(16.8^\circ) = (30.5)$

$a = \frac{30.5}{\tan(16.8^\circ)} = \boxed{101.02}$

$c^2 = a^2 + b^2 \rightarrow c = \sqrt{(101.02)^2 + (30.5)^2}$

$c = \boxed{105.52}$

$A = 90^\circ - 16.8^\circ = \boxed{73.2^\circ}$



13. (6 points) Find the exact value of $\tan[\sin^{-1}(-\frac{3}{5})]$

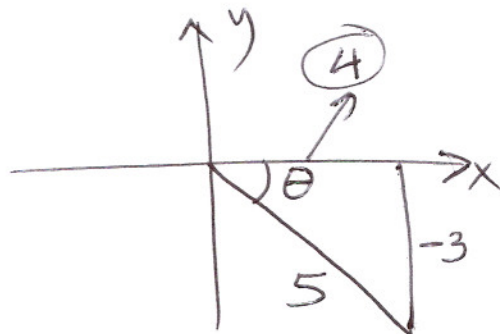
Let $\theta = \sin^{-1}(-\frac{3}{5})$

$\sin \theta = \sin(\sin^{-1}(-\frac{3}{5}))$

$\sin \theta = -\frac{3}{5}$

θ is in QIV

$\tan(\sin^{-1}(-\frac{3}{5})) = \tan \theta = \boxed{-\frac{3}{4}}$



14. (8 points) Find the **exact** value of each expression. Do not use a calculator.

$$(i) \cos 225^\circ = -\cos 45^\circ$$

$$= \boxed{-\frac{\sqrt{2}}{2}}$$

$$(ii) \tan\left(\frac{-\pi}{6}\right) = -\tan \frac{\pi}{6}$$

$$= \boxed{-\frac{\sqrt{3}}{3}}$$

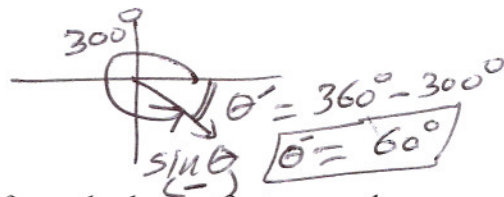
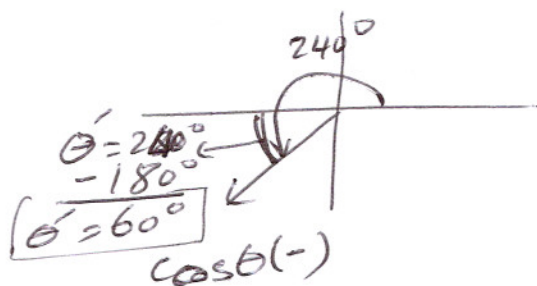
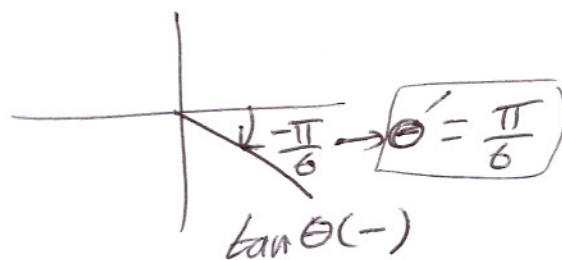
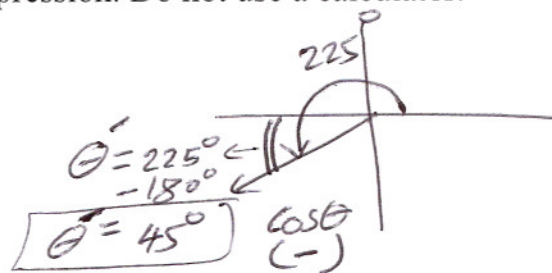
$$(iii) \sec 240^\circ = -\sec 60^\circ$$

$$= -\frac{1}{\cos 60^\circ}$$

$$= \boxed{-2}$$

$$(iv) \sin 300^\circ = -\sin 60^\circ$$

$$= \boxed{-\frac{\sqrt{3}}{2}}$$



15. (6 points) From a point on level ground 80 meters from the base of a tower, the angle of elevation is 85.4° . Approximate the height of the tower to the nearest meter.

$$\tan(85.4^\circ) = \frac{h}{80}$$

$$h = 80 \cdot \tan(85.4^\circ)$$

$$h = 994.3$$

$$\boxed{h \approx 994 \text{ m}}$$

