PRINCE SULTAN UNIVERSITY

Department of Mathematical Sciences

MATH 002 Final Examination

Saturday, 28 January 2005

(051)

Time allowed: 150 minutes

Student Name: Answer Key

Student ID number: ____________________

Section: ____________________

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. If your mobile phone is seen or heard, your exam will be taken immediately.
6. You must show all your work beside the problem. Be organized.
7. You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.
8. This examination has 20 problems, one with two parts. Make sure your paper has all these problems.

Problems	Max points	Student's Points
1,2,3	17	
4,5,6,7	17	
8,9,10	18	
11,12,13	14	
14,15,16	12	
17,18	13	
19,20	9	
Total	100	

1. (6 points) Use a calculator to find the value of the trigonometric functions to four decimal places.

(i) $\sec 55^{\circ} = \boxed{1.7434}$

(ii) $\cot \frac{\pi}{18} = \boxed{5.6713}$

(iii) $\csc 35^{\circ} = \boxed{1.7434}$

2. (6 points) Solve the exponential equation: $e^{6+3x} - 18 = 34$ .Use a calculator to find a decimal approximation, correct to three decimal places.

$$e^{6+3x} - 18 = 34$$

$$e^{6+3x} = 34 + 18 = 52$$

$$\cancel{\ln e}^{6+3x} = \ln 52$$

$$6 + 3x = \ln 52$$

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{(\ln 52) - 6}{3}$$

$$x = \frac{\ln(52) - 6}{3} = \boxed{-0.683}$$

3. (5 points) Find the amplitude and period then graph two periods of $y = 2\sin 2x$.

amplitude, $A = |2| = \boxed{2}$

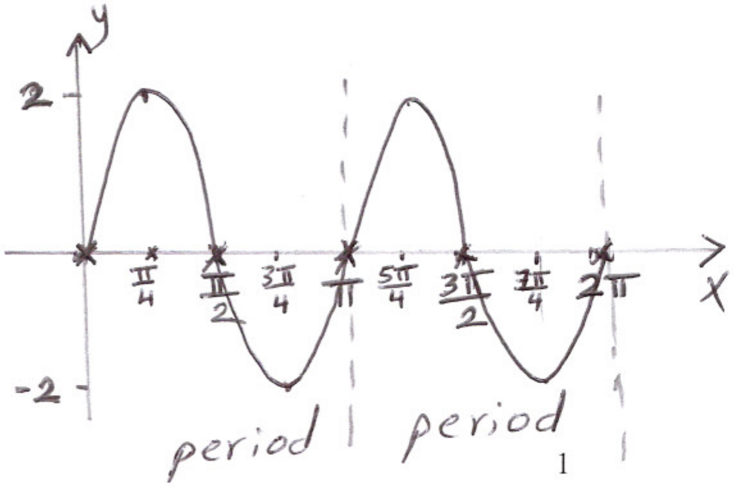
period, $P = \frac{2\pi}{B} = \frac{2\pi}{2} = \boxed{\pi}$

phase shift, $x = 0$

start $\rightarrow x = 0$

End $\rightarrow x = 0 + \pi = \pi$

Middle $\rightarrow x = \frac{0 + \pi}{2} = \frac{\pi}{2}$



4. (6 points) Solve the equation $2\sin^2 x + 7\sin x = 4$ on the interval $[0, 2\pi)$.

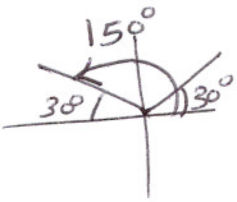
$2\sin^2 x + 7\sin x - 4 = 0$

$(2\sin x - 1)(\sin x + 4) = 0$

$2\sin x - 1 = 0$ or $\sin x + 4 = 0$

$2\sin x = 1$ $\quad$ $\sin x \neq -4$

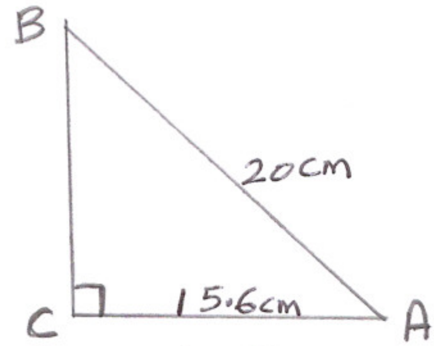
$\sin x = \frac{1}{2}$ (QI, II) $\quad$ not possible



$x = \sin^{-1}\left(\frac{1}{2}\right) = 30°$ (QI, II)

$x = \{30°, 150°\}$

5. (3 points) Find the measure of B in the triangle ABC if $C = 90°$, $b = 15.6$ cm, and $c = 20$ cm. Round to the nearest degree.

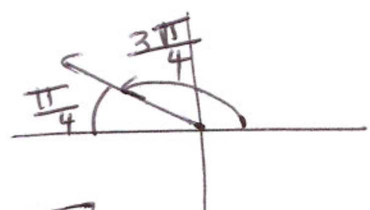


$\sin B = \frac{15.6}{20}$

$B = \sin^{-1}\left(\frac{15.6}{20}\right)$

$B = 51.26° \simeq \boxed{51°}$

6. (4 points) Write $(\cos\frac{\pi}{6})(\cos\frac{7\pi}{12}) - (\sin\frac{\pi}{6})(\sin\frac{7\pi}{12})$ as the sine or cosine of an angle. Then find the exact value of the expression.



$= \cos\left(\frac{\pi}{6} + \frac{7\pi}{12}\right)$

$= \cos\left(\frac{2\pi}{12} + \frac{7\pi}{12}\right)$

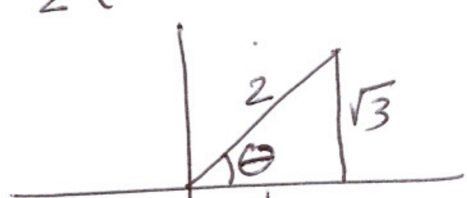
$= \cos\left(\frac{9\pi}{12}\right)$

$= \cos\left(\frac{3\pi}{4}\right) = -\cos\frac{\pi}{4} = \boxed{-\frac{\sqrt{2}}{2}}$

7. (4 points) Find the **exact** value of $\tan(\sin^{-1}\frac{\sqrt{3}}{2})$

Let $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\sin\theta = \sin\left(\sin^{-1}\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$ (QI)

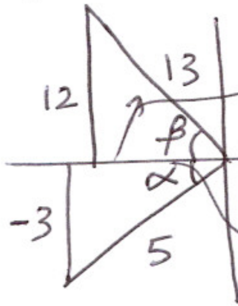


$\tan\theta = \tan\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

$= \frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$

$\sqrt{2^2 - (\sqrt{3})^2}$
$= \sqrt{4-3}$
$= \sqrt{1} = 1$

8. (5 points) Find the **exact** value of $\sin(\alpha+\beta)$ if $\sin\alpha = \frac{-3}{5}$, α lies in quadrant III, and $\sin\beta = \frac{12}{13}$, β lies in quadrant II.



$\sqrt{13^2-12^2} = \sqrt{25} = 5$

$\sqrt{5^2-(-3)^2} = \sqrt{25-9} = \sqrt{16} = 4 = -4$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(-\frac{3}{5}\right)\left(\frac{-5}{13}\right) + \left(\frac{-4}{5}\right)\left(\frac{12}{13}\right)$

$= \frac{15}{65} - \frac{48}{65}$

$= \boxed{-\frac{33}{65}}$

9. (8 points) Verify the following identities:

(i) $\cos(x+\frac{\pi}{2}) = -\sin x$

L.H.S. $\cos(x+\frac{\pi}{2}) = \cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2}$

$= 0 - \sin x\,(1)$

$= \boxed{-\sin x}$

$= R.H.S$

(ii) $(\sec x - \tan x)^2 = \frac{1-\sin x}{1+\sin x}$

L.H.S. $(\sec x - \tan x)^2 = \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2 = \left(\frac{1-\sin x}{\cos x}\right)^2$

$= \frac{(1-\sin x)^2}{\cos^2 x} = \frac{(1-\sin x)(1-\sin x)}{1-\sin^2 x}$

$= \frac{\cancel{(1-\sin x)}(1-\sin x)}{\cancel{(1-\sin x)}(1+\sin x)} = \frac{1-\sin x}{1+\sin x} = R.H.S$

10. (5 points) Graph the solution set of the system of inequalities:

$4x+2y \geq -8 \longrightarrow$ (1)

$2x - y \leq 4 \longrightarrow$ (2)

(1) $4x+2y=-8$

x	y
0	-4
-2	0

$(0,-4)$
$(-2,0)$

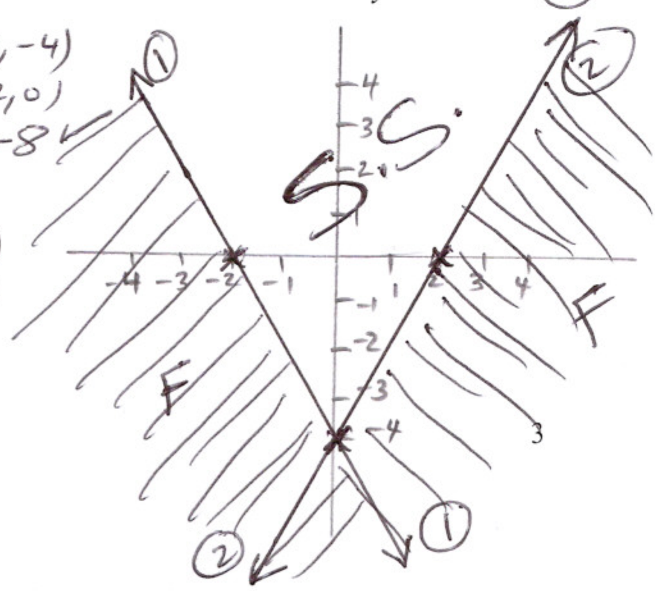
test $(0,0) \longrightarrow 0 \geq -8$

(2) $2x-y=4$

x	y
0	-4
2	0

$(0,-4)$
$(2,0)$

test $(0,0) \longrightarrow 0 \leq 4$



11. (4 points) The sum of four times a first number and twice a second number is -8. If the second number is subtracted from three times the first number, the result is -1. Find the numbers.

x y

$$4x + 2y = -8 \longrightarrow (1)$$
$$2\cdot(\ 3x - y = -1) \longrightarrow (2)$$

$$4x + \cancel{2y} = -8 \longrightarrow (1)$$
$$6x - \cancel{2y} = -2 \longrightarrow (2)$$

$$\frac{10x}{10} = \frac{-10}{10}$$

$$\boxed{x = -1}$$

12. (5 points) Graph $25(x-4)^2 + 4y^2 = 100$ and give the location of its foci. Ellipse

divide by 100 $\longrightarrow \dfrac{(x-4)^2}{4} + \dfrac{y^2}{25} = 1$

center $(4,0)$

ellipse is on y-axis (vertical)

$a^2 = 25 \longrightarrow a = \pm 5$

vertices $(4, 5)$

$(4, -5)$

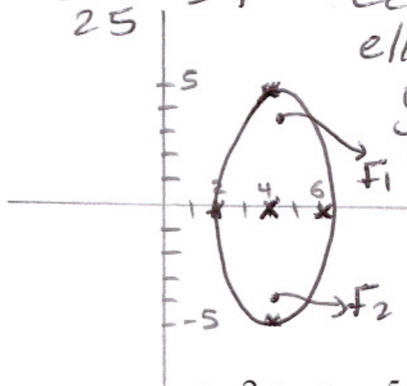
$b^2 = 4 \longrightarrow b = \pm 2$

short ends $(6, 0)$

$(2, 0)$

$c^2 = a^2 - b^2$

$= 25 - 4 = 21 \Rightarrow c = \pm\sqrt{21}$



$F_1(4, \sqrt{21})$

$F_2(4, -\sqrt{21})$

13. (5 points) Use Gaussian elimination to solve the system:
$$\begin{aligned} x - 2y - z &= -5 \\ 2x - 3y - z &= 0. \\ 3x - 4y - z &= 1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 2 & -3 & -1 & 0 \\ 3 & -4 & -1 & 1 \end{array}\right]$$

$$\xrightarrow[-3R_1+R_3]{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 0 & 1 & 1 & 10 \\ 0 & 2 & 2 & 16 \end{array}\right]$$

$$\xrightarrow{-2R_2+R_3} \left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 0 & 1 & 1 & 10 \\ 0 & 0 & 0 & -4 \end{array}\right] \longrightarrow \text{No solution}$$

$$0 \neq -4$$

14. (5 points) Find the standard form of the equation of the hyperbola with the given conditions: foci (-8,0), (8,0); and vertices: (-6,0), (6,0).

center $(0,0)$
horizontal hyperbola

$c = \pm 8 \rightarrow c^2 = 64$

$a = \pm 6 \longrightarrow a^2 = 36$

$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2 = 64 - 36 = 28$

$$\boxed{\frac{x^2}{36} - \frac{y^2}{28} = 1}$$

15. (3 points) Evaluate: $\begin{array}{ccc} + & - & + \end{array}$ $\begin{vmatrix} 3 & 1 & -4 \\ -2 & 0 & 5 \\ 5 & 1 & 3 \end{vmatrix}$

$$-1\begin{vmatrix} -2 & 5 \\ 5 & 3 \end{vmatrix} + \cancel{0\begin{vmatrix} 3 & -4 \\ 5 & 3 \end{vmatrix}} - 1\begin{vmatrix} 3 & -4 \\ -2 & 5 \end{vmatrix}$$

$= -1(-6 - 10) - 1(15 - 8)$

$= 16 - 7$

$= \boxed{9}$

16. (4 points) Solve the following system **for *x* only**, using Cramer's Rule

$3x + y - 4z = -7$

$-2x + 5z = 9$

$5x + y + 3z = -4$

$$D = \begin{vmatrix} 3 & 1 & -4 \\ -2 & 0 & 5 \\ 5 & 1 & 3 \end{vmatrix} = \boxed{9}$$

$$D_x = \begin{vmatrix} -7 & 1 & -4 \\ 9 & 0 & 5 \\ -4 & 1 & 3 \end{vmatrix} = -1\begin{vmatrix} 9 & 5 \\ -4 & 3 \end{vmatrix} + 0\,\cancel{\big|\ \ \big|} - 1\begin{vmatrix} -7 & -4 \\ 9 & 5 \end{vmatrix}$$

$= -1(27 + 20) - 1(-35 + 36)$

$= -47 - 1$

$D_x = -48$

$$x = \frac{D_x}{D} = \boxed{\frac{-48}{9}}$$

17. (5 points) Let $B=\begin{bmatrix}1&-1&1\\0&2&-1\\2&3&0\end{bmatrix}$. Find B^{-1}.

$$\left[\begin{array}{ccc|ccc}1&-1&1&1&0&0\\0&2&-1&0&1&0\\2&3&0&0&0&1\end{array}\right]$$

$$\xrightarrow{-2R_1+R_3}\left[\begin{array}{ccc|ccc}1&-1&1&1&0&0\\0&2&-1&0&1&0\\0&5&-2&-2&0&1\end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_2}\left[\begin{array}{ccc|ccc}1&-1&1&1&0&0\\0&1&-\frac{1}{2}&0&\frac{1}{2}&0\\0&5&-2&-2&0&1\end{array}\right]$$

$$\xrightarrow[-5R_2+R_3]{R_2+R_1}\left[\begin{array}{ccc|ccc}1&0&\frac{1}{2}&1&\frac{1}{2}&0\\0&1&-\frac{1}{2}&0&\frac{1}{2}&0\\0&0&\frac{1}{2}&-2&\frac{-5}{2}&1\end{array}\right]$$

$$\xrightarrow{2R_3}\left[\begin{array}{ccc|ccc}1&0&\frac{1}{2}&1&\frac{1}{2}&0\\0&1&-\frac{1}{2}&0&\frac{1}{2}&0\\0&0&1&-4&-5&2\end{array}\right]$$

$$\xrightarrow[\frac{1}{2}R_3+R_2]{-\frac{1}{2}R_3+R_1}\left[\begin{array}{ccc|ccc}1&0&0&3&3&-1\\0&1&0&-2&-2&1\\0&0&1&-4&-5&2\end{array}\right]$$

B^{-1}

18. (4 points) Let $A=\begin{bmatrix}-2&1&-1\\-5&2&-1\\3&-1&1\end{bmatrix}$, and $B=\begin{bmatrix}1&0&1\\2&1&3\\-1&1&1\end{bmatrix}$ Determine whether B is the multiplicative inverse of A.(Show the answer in details).

$$A\cdot B=\begin{bmatrix}-2&1&-1\\-5&2&-1\\3&-1&1\end{bmatrix}\cdot\begin{bmatrix}1&0&1\\2&1&3\\-1&1&1\end{bmatrix}$$

$$=\begin{bmatrix}-2+2+1&0+1-1&-2+3-1\\-5+4+1&0+2-1&-5+6-1\\3-2-1&0-1+1&3-3+1\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I$$

B is multiplicative inverse of A

19. (8 points) Given that $A=\begin{bmatrix}-2 & 4\\ 3 & -1\\ 1 & -3\end{bmatrix}$, $B=\begin{bmatrix}5 & -1\\ -3 & 2\end{bmatrix}$, and $C=\begin{bmatrix}-2 & 3\\ 4 & -5\end{bmatrix}$. Find

(i) $B+2C=\begin{bmatrix}5 & -1\\ -3 & 2\end{bmatrix}+\begin{bmatrix}-4 & 6\\ 8 & -10\end{bmatrix}=\boxed{\begin{bmatrix}1 & 5\\ 5 & -8\end{bmatrix}}$

(ii) $\underset{2\times 2\ \ 3\times 2}{BA}=$ Not possible

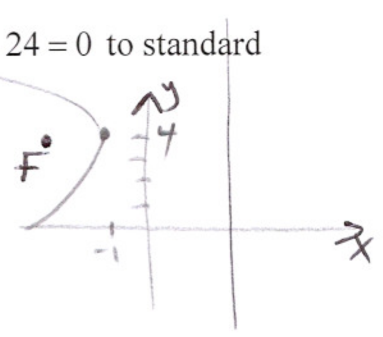
(iii) $BC-2B=\begin{bmatrix}5 & -1\\ -3 & 2\end{bmatrix}\begin{bmatrix}-2 & 3\\ 4 & -5\end{bmatrix}-\begin{bmatrix}10 & -2\\ -6 & 4\end{bmatrix}$

$=\begin{bmatrix}-14 & 20\\ 14 & -19\end{bmatrix}-\begin{bmatrix}10 & -2\\ -6 & 4\end{bmatrix}=\boxed{\begin{bmatrix}-24 & 22\\ 20 & -23\end{bmatrix}}$

(iv) $B^{-1}=\frac{1}{10-3}\begin{bmatrix}2 & 1\\ 3 & 5\end{bmatrix}=\frac{1}{7}\begin{bmatrix}2 & 1\\ 3 & 5\end{bmatrix}$

$=\boxed{\begin{bmatrix}\frac{2}{7} & \frac{1}{7}\\ \frac{3}{7} & \frac{5}{7}\end{bmatrix}}$

20. (5 points) Convert the equation of the parabola $y^2-8y+8x+24=0$ to standard form. Then find the vertex, focus, and directrix.



$(y^2-8y+4^2)+8x+24=0+4^2$

$(y-4)^2=-8x-24+16$

$(y-4)^2=-8x-8$

$\boxed{(y-4)^2=-8(x+1)}$

vertex $(-1,4)$

$4p=-8\rightarrow p=-2$

Focus $\longrightarrow(-1-2,4)=\boxed{(-3,4)}$

directrix $\longrightarrow\boxed{x=1}$