Prince Sultan University

Department of Mathematics and Physical Sciences

Math 225 – Differential Equations

Final Examination

Fall 2015-2016, Term 151

Wednesday, December 24, 2015

Time allowed: 120 minutes — Dr. Bahaaeldin Abdalla

Maximum points: 80 — Section # 130

Student Name: — ID #

1. (16 points in total)

   (a) (9 points) Find the solution of the given initial value problem.

   $$y^{(4)} - y = 0; \qquad y(0) = 0, \quad y'(0) = 0, \quad y''(0) = -2, \quad y'''(0) = 6.$$

   (b) (7 points) Find a particular solution for the differential equation

   $$y^{(4)} - y = e^{-t} + t.$$

2. (12 points in total) Let x, x^2 and x^3 be the solutions of the homogeneous equation corresponding to

   $$x^3 y''' - 3x^2 y'' + 6xy' - 6y = 2x, \quad x > 0.$$

   (a) (8 points) Determine a particular solution.

   (b) (2 points) Verify that the particular solution you obtained in (a) satisfy the nonhomogeneous differential equation.

   (c) (2 points) Write the general solution of the nonhomogeneous differential equation.

3. (13 points in total) Let $y = \sum_{n=0}^{\infty} a_n x^n$ be a series solution of $(1-x)y'' + y = 0$ about $x_0 = 0$.

   (a) (7 points) Find the recurrence relation that describes the coefficients a_n.

   (b) (6 points) Find the first three terms in each of two solutions y_1 and y_2.

4. (12 points in total)

(a) (3 points) Show that $x=0$ is a regular singular point of the differential equation

$$4x^2y''+8xy'+17y=0.$$

(b) (9 points) Find the solution of the initial value problem

$$4x^2y''+8xy'+17y=0, \qquad y(1)=2, \quad y'(1)=2.$$

5. (7 points) Express the solution of the given initial value problem in terms of a convolution integral.

$$y''+2y'+y=\sin\alpha t; \qquad y(0)=0, \quad y'(0)=0.$$

6. (12 points) A fundamental matrix of the system $\mathbf{x}'=\begin{pmatrix}1 & 4\\ 1 & -2\end{pmatrix}\mathbf{x}$ is $\boldsymbol{\psi}(t)=\begin{pmatrix}-e^{-3t} & 4e^{2t}\\ e^{-3t} & e^{2t}\end{pmatrix}$.

Use this information to find the general solution of $\mathbf{x}'=\begin{pmatrix}1 & 4\\ 1 & -2\end{pmatrix}\mathbf{x}+\begin{pmatrix}e^{3t}\\ 2e^{t}\end{pmatrix}$.

7. (8 points) Suppose that the system $\mathbf{x}'=\mathbf{A}\mathbf{x}$ has the following eigenvalues and corresponding eigenvectors:

$$\lambda_1=-1 \quad \boldsymbol{\xi}^{(1)}=\begin{pmatrix}-1\\ -1\\ 1\end{pmatrix}, \quad \lambda_2=-2+2i \quad \boldsymbol{\xi}^{(2)}=\begin{pmatrix}1+i\\ -1+i\\ 2\end{pmatrix} \quad \text{and} \quad \lambda_3=-2-2i \quad \boldsymbol{\xi}^{(3)}=\begin{pmatrix}1-i\\ -1-i\\ 2\end{pmatrix}.$$

Express the general solution of the system in terms of real-valued functions.

Good Luck