Prince Sultan University

Deanship of Educational Services

Department of Mathematics and General Sciences

COURSE DETAILS:

LINEAR ALGEBRA	MATH 223	MAJOR EXAM II
Semester:	Spring Semester --Term 182	
Date:	Sunday April 7^{th}, 2019	
Time Allowed:	90 minutes	

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section:	730
Instructor's Name:	Dr. Jamiiru Luttamaguzi

INSTRUCTIONS:

- **Start your working immediately below the problem and <u>continue to use the back</u>** of the page for extra space.
- You may use a scientific calculator that does not have programming or graphing capabilities.
- **NO borrowing** calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work where needed and be organized.

GRADING:

	Page 2	Page 3	Page 4	Page 5	Page 6	Total	Total
Question	**1**	**2**	**3**	**4**	**5**	**Out of 55**	**Out of 25**
Marks	**14**	**14**	**10**	**10**	**7**	**55**	**25**
Grade							

Question 1 [14 points]: Take the matrix $A = \begin{bmatrix} -6 & 2 \\ 4 & 1 \end{bmatrix}$

(a) Find the eigenvalues and bases of the eigenspaces of each eigenvalue for A.
(b) What is the invertible matrix P and diagonal matrix D such that $AP = PD$?
(c) What are the eigenvalues and bases of the eigenspaces for A^4.

Start working below:

Question 2 [14 points]: Answer each of the following

(a) **Write the standard matrix** to rotate about the origin by 30 degrees in R^2. What is the effect (image) of rotating a line segment joining points $B(-2,2)$ and $C(4,0)$. Draw the original line segment and the rotated line segment.

(b) Using **a linear transformation standard matrix,** find the reflected triangle in the x-axis R^2 of the triangle with vertices$A(1,1)$, $B(3,2)$ and $C(4,0)$. [Do not draw, Give the vertex coordinates of the reflected triangle]

Start working below:

Question 3 [10 points]: Use the Wronskian to show that $f_1(x) = x^2 + 1$, $f_2(x) = x^2 - 1$, and $f_3(x) = x + 1$ are linearly independent. Then write $p(x) = ax^2 + bx + c$ as an explicit linear combination of f_1, f_2 and f_3 (this shows that these functions span P_2).
Start working below:

Question 4 [10 points]: Let V be the vector space of 2×2 matrices and W the subspace of V such that W: Set of matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $\text{trace}(A) = a + d = 0$.

(a) Verify that W is a subspace of V.

(b) Show that $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \right\}$ is a basis for W.

(c) Find a matrix B whose coordinate vector is $[B]_S = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$.

Start working below:

Question 5 [7 points]: Suppose $B_1 = \{\boldsymbol{u_1}, \boldsymbol{u_2}\}$ and $B_2 = \{\boldsymbol{v_1}, \boldsymbol{v_2}\}$ are basis vectors for $V = R^2$.

Suppose that $\boldsymbol{u}_1 = (1,2)$, $\boldsymbol{u}_2 = (2,3)$ and a transition basis matrix is $P_{B_1 \to B_2} = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$.

(a) What is the coordinate vector $[w]_{B_2}$ of $w = (0,1)$ with respect to B_2?

(b) What is the transition matrix $P_{B_2 \to B_1}$

Start working below: