Prince Sultan University

Deanship of Educational Services
Department of Mathematics
and General Sciences

COURSE DETAILS:

LINEAR ALGEBRA MATH 223 MAJOR EXAM I	
Semester:	Spring Semester --Term 182
Date:	Wednesday February 20th, 2019
Time Allowed:	90 minutes

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section:	730
Instructor's Name:	Dr. Jamiiru Luttamaguzi

INSTRUCTIONS:

- Continue **to use the back** of the pages for extra space of the same the paper of the problem.
- You may use a scientific calculator that does not have programming or graphing capabilities.
- **NO borrowing** calculators.
- **NO** talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work where needed and be organized.

GRADING:

	Page 2	Page 3	Page 4	Page 5	Total	Total
Questions	**1-7**	**8**	**9**	**10**	**Out of 80**	**Out of 25**
Marks	**22**	**14**	**11**	**33**	**80**	**25**
Grade						

Part I: Short Answer Questions [18 points, 3 points each]

1. Suppose the matrices have indicated dimensions: $A(4 \times 2), B(4 \times 4), C(4 \times 2),$ and $D(2 \times 4)$. The dimension of $A^T(B + (CD)^{-1})$ is _____.

2. Suppose matrices A and B have dimension 3×3 and $\det(A) = 2, \det(B) = 3$. Then $\det(2A^{-1}B^T) =$ _____.

3. Suppose the equation below has $A, B,$ and C as $n \times n$ invertible matrices. Solve the equation below for Y.

$$A^{-1}YB = C$$

The solution $Y =$

4. Suppose $\boldsymbol{a}$ and $\boldsymbol{b}$ are vectors in R^2, $\boldsymbol{c}$ is a vector in R^3 and k a real scalar. Which of the following operations make sense (circle all that apply)

(i) $\boldsymbol{a} \cdot \boldsymbol{b} + k$ (ii) $k\|\boldsymbol{a}\|$ (iii) $\|\boldsymbol{b} - \boldsymbol{a}\| + \|\boldsymbol{c}\|$

5. The distance between the plane $2x - 4y + z = 2$ and the origin is _____.

6. The diagonal matrix D for which the matrix product is true

$$\begin{bmatrix} 2 & -4 & 6 \\ -6 & 2 & 4 \\ 4 & -2 & 4 \end{bmatrix} \times D = \begin{bmatrix} 2 & -8 & 18 \\ -6 & 4 & 12 \\ 4 & -4 & 12 \end{bmatrix}$$

is $D =$ and $D^k =$

Part II: Detailed Answer Questions (Show working) [62 points]

7. [4 points] Take the matrix $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$. What is $A + 4(A^{-1})$?

8. [3+5+5+1 = 14 points] Take the matrix below

$$A = \begin{bmatrix} -3 & 5 & 7 \\ 0 & 1 & 2 \\ -1 & 1 & 4 \end{bmatrix}$$

(a) What is $A + A^T$ and $\text{trace}(A + A^T)$?

(b) Evaluate the determinant of A using the cofactor method.

(c) Evaluate the determinant of A using row reduction.

(d) Is the matrix A invertible?

[If needed, continue working at the back of this page]

9. [1+5+1+1+1+2 = 11 points] Take the linear system below

$$\begin{cases} w + 5x + 9y - z = 4 \\ 2w + 2x - 2y + 6z = 0 \end{cases}$$

(a) Is the system homogeneous?
(b) Solve the system using the Gaussian-Jordan elimination method.
(c) What are the free variables?
(d) What are the leading variables?
(e) What are the pivot positions?
(f) Write the system above in form of matrix product in the form $AX = B$.

[If needed, continue working at the back of this page]

10. [2+5+5+3+3+4+4+4+3 = 33 points] Define vectors $\boldsymbol{v} = (-2,2)$ and $\boldsymbol{w} = (3,2)$.
    (a) Evaluate $\boldsymbol{a} = \boldsymbol{v} + 3\boldsymbol{w}$.
    (b) Find the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$. Are the two vectors orthogonal?
    (c) Evaluate $\boldsymbol{b} = proj_{\boldsymbol{v}}\boldsymbol{w}$.
    (d) Find the unit vector parallel but opposite to $\boldsymbol{w}$.
    (e) Verify Cauchy's inequality $|\boldsymbol{v} \cdot \boldsymbol{w}| \leq \|\boldsymbol{v}\|\|\boldsymbol{w}\|$
    (f) Draw $\boldsymbol{v}$ and $\boldsymbol{w}$ at the origin and draw the vector $\boldsymbol{u} = \boldsymbol{v} - 2\boldsymbol{w}$ at the origin.
    (g) What is $\|5\boldsymbol{v}\| - d(\boldsymbol{v}, \boldsymbol{w})$?
    (h) What is the terminal point B of vector $\boldsymbol{v}$ if its initial point is set to point $A = (1,1)$?
    (i) For what of x is the vector $\boldsymbol{u} = (x, -3)$ parallel to vector $\boldsymbol{w}$?

[If needed, continue working on back pages]