Prince Sultan University

Deanship of Educational Services

Department of Mathematics
and General Sciences

COURSE DETAILS:

LINEAR ALGEBRA MATH 223 FINAL EXAM	
Semester:	Spring Semester --Term 182
Date:	Saturday April 20th, 2019
Time Allowed:	180 minutes

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section:	730
Instructor's Name:	Dr. Jamiiru Luttamaguzi

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities.
- NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

Page	2	3	4	5	6	Total	Total
Questions	1, 2	3, 4	5, 6	7	8		
Marks	16	12	11	9	12	60	40
Grade							

Question 1: [8 points] Answer each of the 8 questions below. **[Just circle the answer]**

(a) A single equation with two or more unknowns must always have infinitely many solutions. (True/False)

(b) All leading 1's in a matrix in row echelon form must occur in different columns. (True/False)

(c) For all matrices A and B: $(A+B)^2 = A^2 + 2AB + B^2$. (True/False)

(d) If A and B are $n \times n$ matrices with A invertible, then $\det(A^{-1}BA) = \det(B)$. (True/False)

(e) Every vector space is a subspace of itself. (True/False)

(f) The set R^2 is a subspace of R^3. (True/False)

(g) There is a set of 11 vectors in R^{17} that span R^{17}. (True/False)

(h) There is a set of 11 linearly independent vectors in R^{17}. (True/False)

Questions 2 to 8: Show your working

Question 2: [2+2+4 = 8 points] Take the matrix $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

(a) What is: $A + 4(A^{-1})$?

(b) What is: $\text{trace}(A^{-1})$?

(c) Let $\langle X, Y \rangle = \text{trace}(X^T Y)$. Verify that $\langle B, B \rangle = 0$ **if and only if** $B = \underline{0}$ (zero matrix on $M_{2\times 2}$).

Solution below:

Question 3: [3 points] Let A be a matrix such that: $A^2 = A$. Define $B = I - A$, show that $B^2 = B$.
Solution below:

Question 4: [3+3+3 points] Let $S = \{(1,1,1), (2,2,2), (1,2,3), (2,4,6)\}$ and $T = \{(1,1,1), (1,2,3)\}$.
(a) Show that the set S does not span R^3 using the definition.
(b) Show that the set T is linearly independent using the definition.
(c) Find basis of the orthogonal compliment $W^\perp$ of the subspace $W = \text{span}(T)$.
Solution below:

Question 5: [3+1+2 = 6 points] Find the basis S and dimension for the subspace W in R^4 below:

$$W = \{(a, b, c, d):\ c = a + b,\ \ d = a - b\}$$

and find the coordinate vector with respect to the basis S you got of vector $w = (4, 50, 54, -46)$.

Solution below:

Question 6: [3+2 = 5 points] Let $V = M_{2\times 2}$. Show that W the set of matrices of the type $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ forms a subspace of V but the set U of matrices of the type $\begin{bmatrix} a & b \\ 1 & 0 \end{bmatrix}$ is **not** a subspace of V.

Solution below:

Question 7: [9 points] Solve the system

$$y_1' = y_1$$
$$y_2' = -2y_1 - 4y_2$$

with initial conditions $y_1(0) = 10$ and $y_2(0) = 5$.

Solution below:

Question 8: [3+5+4 = 12 points] Define vectors g and h as $g(x) = 3x^2$ and $h(x) = x + 1$ on the inner product space P_2 with inner product: $\langle g, h \rangle = \int_0^1 g(x)h(x)dx$

(a) Find the distance $d(g, h) = \|g - h\|$

(b) Find the angle in degrees between the vectors g and h.

(c) Find an orthogonal projection of g onto h.

Solution below: