Prince Sultan University

Deanship of Educational Services

Department of Mathematics
and General Sciences

COURSE DETAILS:

LINEAR ALGEBRA MATH 223 MAJOR EXAM II	
Semester:	Spring Semester --Term 172
Date:	Sunday April 22nd, 2018
Time Allowed:	90 minutes

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section:	157
Instructor's Name:	Dr. Jamiiru Luttamaguzi

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

	Page 1	Page 2	Page 3	Page 4	Total	Total
Questions	1,2	3,4	5	6		
Marks	11	11	12	11	45	25
Grade						

1. [8 points] Short Answers: (Just give answer, no partial credit)
   (i) The dimension of the vector space of n-degree polynomials P_n is: ____________.

   (ii) The dimension of the vector space of 2 by 3 matrices M_{2x3} is: ______________.

   (iii) The characteristic polynomial of the standard matrix of the linear transformation $T(x, y) = (2y, 2x)$ is _______________________________.

   (iv) The rows of a matrix A whose det(A) = 4 are (select only one)
   (a) Linearly Dependent (b) Linearly Independent

   (v) Does the vector $(4,0,6)$ belong to $\text{span}(\{1,0,0), (2,0,3)\}$? (select only one) (a) Yes (b) No.

   (vi) The set $W = \{\text{A functons} f \text{ in } C(-\infty,\infty) : \text{such that} f(0) = 0\}$ is a subspace of $C(-\infty,\infty)$.
   (select only one) (a) Yes (b) No

   (vii) Take the basis $B = \{(1,1,1), (0,-1,-1)\}$. The vector v whose coordinate vector is $(v)_B = (2,2)$ is v = ______________________

   (viii) Take $< f, g >= \int_0^1 f(x)g(x)dx$ and $p(x) = 8x$. The distance $d(p, 2p)$ is ____________

2. [3 points] What is the standard matrix M in R^2 of the operation: A projection on to the x-axis followed by a rotation clockwise around the origin by 45° (degrees)?

3. [3+1+3+1 = 8 points] Take two linear transformations $T_1, T_2 : R^2 \to R^2$ defined by $T_1 : w_1 = 4x_1 + 3x_2,\ w_2 = x_1 - x_2$ and $T_2 : w_1 = 24x_1,\ w_2 = 24x_2$.

(a) Write down the matrix representation $[T_1^{-1}]$.

(b) Find the image under T_1^{-1} of the point $(7,7)$.

(c) What is the matrix representation of $[T_1^{-1} \circ T_2]$?

(d) Find $(T_1^{-1} \circ T_2)(0,1)$.

4. [3 points] Find the value(s) of x that make the set $S = \{(1,4,0), (x,0,1), (4,0,x)\}$ linearly independent.

5. [3+2+2+5 = 12 points] Let $B = \{u = (1,2,3), v = (4,10,16)\}$ be a basis for a subspace W in R^3 using the standard inner product $< u, v > = u_1v_1 + u_2v_2 + u_3v_3$.

(a) What is $\|u\| = ?, \|v\| = ?$, and $< u, v >= ?$

(b) Use your answers in (a) to compute $< u + 2v, -u + 3v >$

(c) What is the angle in degrees between *u* and *v*?

(d) What is the basis of the orthogonal compliment $W^{\perp}$ of W ?

6. [11 points] Solve the differential equation system with the given initial conditions:

$$\begin{cases} x_1' = -7x_1 - 6x_2 \\ x_2' = 15x_1 + 12x_2 \end{cases}$$, with $x_1(0) = 1$ and $x_2(0) = 1$.