Prince Sultan University

Deanship of Educational Services
Department of Mathematics
and General Sciences

COURSE DETAILS:

LINEAR ALGEBRA	MATH 223	MAJOR EXAM I
Semester:	Spring Semester --Term 172	
Date:	Wednesday March 7th, 2018	
Time Allowed:	90 minutes	

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section:	157
Instructor's Name:	Dr. Jamiiru Luttamaguzi

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

	Page 1	Page 2	Page 3	Page 4	Total	Total
Questions	**1**	**2,3**	**4,5**	**6,7**		
Marks	**13**	**10**	**11**	**16**	**50**	**25**
Grade						

1. [6+2+2+3] Take the linear system matrix augmented form below in terms of $v, w, x, y,$ and z.

$$\begin{bmatrix} 2 & 4 & 0 & 0 & 0 & 8 \\ -2 & -4 & 1 & 0 & 5 & -10 \\ 2 & 4 & 0 & 1 & 3 & 10 \end{bmatrix}$$

(a) Reduce the matrix augmented form above to Reduced Row Echelon Form (RREF).
(b) What are the pivot positions?
(c) What are the free variables?
(d) Solve the system.

2. [3+3] Consider the linear system in terms of x, y, and z .

$$\begin{cases} x + 4z = 9 \\ ax + 2y = 9 \\ 3x + 3y = 10 \end{cases}$$

(a) Suppose the determinant of the coefficient matrix is 24, find the value of a?

(b) Use Cramer's rule to find the value of x.

3. [2+2] Suppose matrices A and B are given below:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, B = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$$

(a) Compute $A + A^{-1}$

(b) Write down the column-row expansion of $B^T A^T$

4. [2+2+3] Suppose the determinants of 3×3 square matrices A and B are 4 and 5 respectively.
   (a) What is $\det(2A^T)$?
   (b) What is $\det(2A^{-1})$?
   (c) Suppose $f(x,y)=2x^2y^{-3}$. What is $\det(f(A,B))$?

5. [4] Find the orthogonal projection of the vector $u=(3,8)$ to the normal vector of the line $4x+3y=5$ in R^2. What is the length of this orthogonal projection?

6. [3] Find the distance between the point $P(3,8)$ and the line $4x+3y=5$ in R^2.

7. [3+3+1+4+2] Let $u=(1,-5,4,0)$, $v=(2,1,-1,1)$, and $w=(0,1,0,-1)$ be vectors in R^4.

   (a) Evaluate $\|\|u-2v\|w\|$.

   (b) Evaluate $\|(u\cdot v)w\|$.

   (c) Are vectors *v* and *w* orthogonal or not orthogonal? Give the reason.

   (d) Evaluate the vector component of *u* perpendicular to *v*.

   (e) Find the value(s) of k such that the vector $a=(k^2,k,1,0)$ is orthogonal to the vector u.