Prince Sultan University

Deanship of Educational Services

Department of Mathematics
and General Sciences

COURSE DETAILS:

LINEAR ALGEBRA	MATH 223 FINAL EXAM
Semester:	Spring Semester --Term 172
Date:	Tuesday May 8th, 2018
Time Allowed:	180 minutes

STUDENT DETAILS:

Student Name:	
Student ID Number:	
Section:	157
Instructor's Name:	Dr. Jamiiru Luttamaguzi

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

GRADING:

Page	1	2	3	4	5	6	7	Total	Total
Question	1	2	3	4	5	6	7		
Marks	10	11	11	18	8	11	11	80	40
Grade									

1. [2+5+3 pts.] Take the **augmented form** in Echelon Form of a homogeneous system $Ax = 0$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

(a) What are its pivot columns?

(b) Assign variables to the system and solve it.

(c) What is the basis and dimension of the solution space?

2. [3+5+3 pts.] Let matrix A be defined as

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 7 & -5 \\ -3 & -1 & -2 \end{bmatrix}$$

(a) Compute $\det(A)$ using the cofactor expansion method.

(b) Compute $\det(A)$ using the row reduction method.

(c) What is $\det(3A^{-2})$? Do not approximate your answer.

3. [3+6+2 pts.] Define an inner product on P_2 as $< p,q > = \int_0^1 p(x)q(x)dx$ and let p, q in P_2 be

$$p(x) = 12x,\ q(x) = 6x^2.$$

(a) Compute $< p,q >$

(b) Compute the angle in degrees between p and q (to 1 decimal). Are p and q orthogonal?

(c) Use the Wronskian to show p and q are linearly independent.

4. [4+5+2+7 pts.] Let $W = \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} : a, b \text{ are real numbers} \right\}$ be a subset of $M_{2\times 2}$.

(a) Show that W is a subspace of $M_{2\times 2}$.

(b) Take set $S = \left\{ C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}$. Show that S is a basis for W.

(c) Write $F = \begin{bmatrix} \frac{1}{2} & -\frac{2}{3} \\ \frac{2}{3} & \frac{1}{2} \end{bmatrix}$ as a linear combination of basis elements C and D.

(d) Define an inner product on $M_{2\times 2}$ as $< A, B > = \text{trace}(A^T B)$. With this inner product the basis S is orthogonal (do not show this).

Take a matrix $G = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$. Find the projection $\text{proj}_W(G)$ of G to W using basis S.

5. [2+2+4 pts.] Consider the standard inner product on R^2. Take the standard matrix M in R^2 of the transformation that reflects in the x-axis and the standard matrix N in R^2 of the transformation that rotates counterclockwise by 90°.

(a) Write down M. Is M an orthogonal matrix or not? Justify?

(b) Write down N. Is N an orthogonal matrix or not? Justify?

(c) Let $v = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, and $w = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$. Compute the 3 distances $d(v, w), d(Mv, Mw)$, and $d(Nv, Nw)$.

6. [11 pts.] Hooke's Law states that the force y applied to a uniform spring is a linear function of the length x of the spring. That is $y = a + bx$. Measurements with errors are done as follows

Weight, y in pounds	0	1	2	3
Length x in inches	−1.4	0.9	3.7	6.2

Find the least squares straight line fit $y = a + bx$ to the data and use it to approximate the spring constant b to 1 decimal place.

7. [1+3+5+2 pts.] Let A be the matrix $A = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$.

(a) What are the eigenvalues of A?

(b) Show that the matrix $P = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$ diagonlizes the matrix A.

(c) Compute and simplify A^{10} **using *diagonalization***.

(d) Does P orthogonally diagonalize A? Why or why not?