Prince Sultan University
Orientation Mathematics Program

MATH 223

Class Major Test II

Semester II, Term 142

Wednesday, May 6th, 2015

Time Allowed: **90** minutes

Student Name: ____

Student ID #: ____

Important Instructions:

1. Once you start the exam, there are no bathroom breaks.
2. You may use a scientific calculator that does not have programming or graphing capabilities.
3. You may NOT borrow a calculator from anyone.
4. You may NOT use notes or any textbook.
5. There should be NO talking during the examination.
6. No usage of phone during exams. Turn it off before starting the exam.
7. Your exam will be taken immediately if your mobile phone is seen or heard.
8. Looking around or making an attempt to cheat will result in your exam being cancelled.

Problems	Max points	Student's Points
1	8	
2	12	
3	15	
4	8	
5	6	
6	8	
7	18	
Total	**75**	

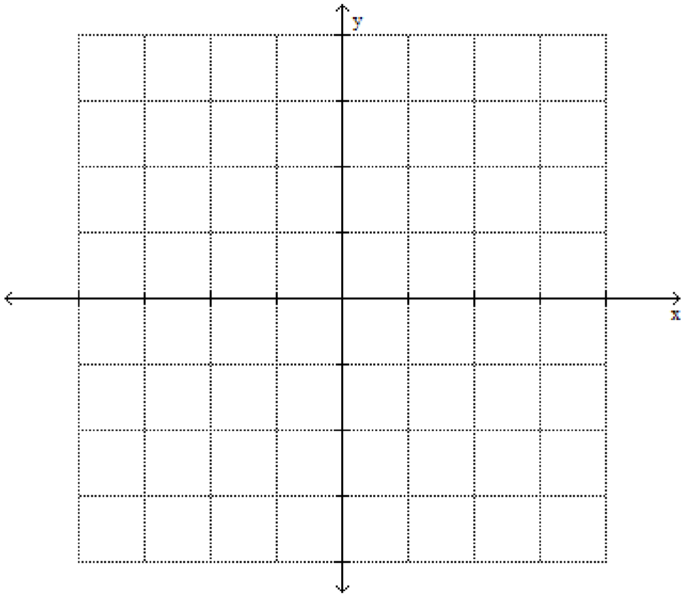
1. (8 points) Let $u = (3i, 1-4i), v = (1-i, 2i)$. Given the equation $2x - 3iu = \overline{v}$ where a complex vector $x = (a, b)$ is in C^2, find a and b.

2. (12 points) A triangle P in R^2 has corners $C_1(3,-3)$, $C_2(0,0)$ and $C_3(3,0)$.

(a) Write a transformation matrix A that rotates points in R^2 about the origin by 45°, and the transformation matrix B that that contracts points by $\frac{1}{3}$.

(b) Use matrices A, B to get the image of the corners when transformed by A followed by the transformation with matrix B. (approximate answer to 2 decimal place)

(c) Draw the original set of vertices P and the final transformed set Q of vertices and shade the resulting triangles.



3. (15 points) Consider the set P_2 of polynomials of degree 2. Let W be a subset of P_2 defined as follows: $W = \{p(x) = ax^2 + bx + c : p(0) = 0\}$ which can also be written as $W = \{p(x) = ax^2 + bx : a, b \in R\}$.

(a) Show that W is a subspace of P_2.

(b) Use a definition or Wronskian to show that the set $B = \{2x^2 + x, x^2 + x\}$ is linearly independent and that $C = \{2x^2 + 2x, x^2 + x\}$ is not linearly independent.

(c) Find the coordinate vector of $p(x) = 4x^2 - 9x$ with respect to B.

4. (8 points) Given a matrix $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 3 & 2 \\ 0 & 1 & 2 \end{bmatrix}$. Show that the columns of matrix A are linearly dependent:

(a) Using the definition.

(b) Using the determinant.

5. (6 points) Suppose R^4 has the usual inner product and W is a subspace with basis $B = \{(1-4,0,0),(0,0,1,0)\}$. What is the basis of the orthogonal complement $W^{\perp}$ and its dimension?

6. (8 points) Consider the inner product space M^2 of 2×2 matrices with the inner product $<A,B>= \text{tr}(A^T B)$. Let $A = \begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 5 \\ -1 & 2 \end{bmatrix}$. Compute the following

(a) $\| A \|$

(b) $d(A,B)$

7. (18 points) Given the matrix $A = \begin{bmatrix} 4 & 5 \\ 4 & 3 \end{bmatrix}$. Answer (a)-(d)

(a) Find the eigenvalues and bases for the corresponding eigenspaces of the matrix A.

(b) Write the matrix D, the matrix P that diagonalizes A and find P^{-1}.

(c) Use the diagonalization to find A^{10}. Do not simplify.

(d) Use the diagonalization to solve the system of ODEs

$y_1' = 4y_1 + 5y_2$

$y_2' = 4y_1 + 3y_2$

with initial conditions $y_1(0) = 1$, $y_2(0) = 0$.

BONUS: Suppose V is an inner product space and $\|u\| = \sqrt{5}$, $\|v\| = 2\sqrt{2}$, and $<u, v> = 2$. Find $\|u - v\|$?