Prince Sultan University
Orientation Mathematics Program

MATH 223

Class Major Test I

Semester II, Term 142

Tuesday, March 17th, 2015

Time Allowed: **90** minutes

Student Name: _____

Student ID #: _____

Important Instructions:

1. Once you start the exam, there are no bathroom breaks.
2. You may use a scientific calculator that does not have programming or graphing capabilities.
3. You may NOT borrow a calculator from anyone.
4. You may NOT use notes or any textbook.
5. There should be NO talking during the examination.
6. No usage of phone during exams. Turn it off before starting the exam.
7. Your exam will be taken immediately if your mobile phone is seen or heard.
8. Looking around or making an attempt to cheat will result in your exam being cancelled.

Problems	Max points	Student's Points
1	14	
2	08	
3	17	
4	10	
5	15	
6	11	
Total	**75**	

1. Consider the matrices

$$A=\begin{bmatrix}2 & 4 & x\\ 1 & 0 & 5x\end{bmatrix} \text{ and } B=\begin{bmatrix}2 & 3x\\ -2 & 5\\ 0 & -1\end{bmatrix}.$$

(a) Compute the determinant of $C=AB$

(b) Compute the trace: $\text{tr}(C)$

(c) Is the matrix $BA+5I_3$ a square matrix.? Why or why not?

(d) Compute the inverse C^{-1}

2. Assume that a system in 5 variables v, w, x, y, z has been reduced to a row reduced augmented form:

$$\begin{bmatrix}1 & 0 & 0 & 4 & 4 & 7\\ 0 & 1 & 1 & 1 & 0 & 6\end{bmatrix}$$

(a) What are the free variables?

(b) Solve the system?

3. Solve the system of equations for *x*, *y*, and *z* using an augmented matrix and **row reduction**. Also solve the system for only z using **Cramer's rule**.

$x+3z=2$

$y+4z=1$

$x+2y+7z=0$

4. Let plane D have equation $3x-4y+z=1$ and plane E have equation $6x-8y+2z=3$.

(a) Show that plane D is parallel to plane E.

(b) Obtain any point P that is on D.

(c) Use (b) to compute the distance between plane D and plane E.

5. Let vectors $u=(6,1)$ and $a=(3,-9)$.

(a) Why is u not orthogonal to a?

(b) What is the angle between u and a in radians to 2 decimal places?

(c) Compute $\|\text{proj}_a u\|$.

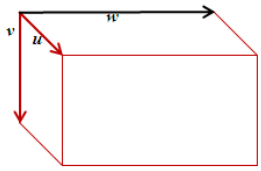
(d) Get the vector component of u orthogonal to a.

6. Let vectors $u=(4,2,0)$ and $v=(3,-1,1)$.

(a) Compute $\|u\|-2\|v\|$ and $\|u\|+\|-2v\|$.

(b) Compute vector w defined by $w=(2u)\times(-3v)$.

(c) Compute the volume of the parallelepiped determined by u,v and w. (It is not necessarily a box!!)



BONUS Short answer questions [6 points]

(a) What elementary matrix of size 4×4 is obtained when 5 times row 2 is added to row 3?

(b) Let u, v be vectors in R^3. Circle the expressions that make sense:

$|u\cdot v|$, $|u\times v|$, $\|u\cdot v\|$, $\|u\times v\|$

(c) Let u ,v, and w be vectors in R^3. Is $(u\cdot v)+w\cdot(u\times w)$ is meaningful? Why or why not?