Prince Sultan University
Orientation Mathematics Program

MATH 223

Final Exam

Semester II, Term 142

Tuesday, May 26th, 2015

Time Allowed: **120** minutes

Student Name: ____

Student ID #: ____

Important Instructions:

1. Once you start the exam, there are no bathroom breaks.
2. You may use a scientific calculator that does not have programming or graphing capabilities.
3. You may NOT borrow a calculator from anyone.
4. You may NOT use notes or any textbook.
5. There should be NO talking during the examination.
6. No usage of phone during exams. Turn it off before starting the exam.
7. Your exam will be taken immediately if your mobile phone is seen or heard.
8. Looking around or making an attempt to cheat will result in your exam being cancelled.

Problems	Max points	Student's Points
1	9	
2	6	
3	6	
4	14	
5	6	
6	16	
7	13	
8	10	
Total	**80**	

1. **[9]** Answer True or False

(a) [   ] If S is a linearly independent set then it is orthogonal set.

(b) [   ] The volume of a parallelepiped made by three vectors $u, v,$ and w is $|u \times (v \times w)|$

(c) [   ] The values of determinants $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ and $\begin{vmatrix} d & e & f \\ -a & -b & -c \\ g & h & i \end{vmatrix}$ are the same.

(d) [   ] If A is invertible, then the reduced row echelon form of A is I_n

(e) [   ] For any square matrix $\det(A+B) = \det(A) + \det(B)$

(f) [   ] All square matrices with distinct eigenvalues and diagonalizable.

(g) [   ] For any square matrix: $(A^T)^{-1} = (A^{-1})^T$

(h) [   ] The inverse of an invertible upper triangular matrix is also upper triangular.

(i) [   ] The vector space: $\text{span}\{(1,0,1),(0,1,0),(1,1,1)\}$ has dimension 2.

2. **[2+4]** Consider a vector $a = (-2,3,7)$ and a point $P(2,0,-2)$.

(a) Find in standard form an equation of a plane through P with normal a.

(b) Find the shortest distance from $P(1,1,1)$ to the plane in (a) above.

3. **[4+2]** Let S and T linear transformations: $S(x, y) = (2y, 2x)$ and $T(x, y) = (x + y, 2x)$.

(a) Write a formula for the composite $T \circ S$ and its matrix representation.

(b) **Use (a)** to get $(T \circ S)(1,-2)$

4. **[8+3+3]** Let a linear operator $L: R^2 \to R^2$ such that $L(1,1) = (1,3)$ and $L(1,-1) = (3,1)$

(a) Find a general formula for $L(x, y)$.

(b) Find the value of $L(4,-5)$.

(c) Write the matrix representation of L.

5. **[6]** Let $L_1 : R^2 \to R^2$ be a linear transformation that dilates points 3 times in the *x*-direction and 4 times in the *y*-direction. Let $L_2 : R^2 \to R^2$ be a linear transformation that projects points onto the *y*-axis. Write down the matrix representations of the transformations L_1 and L_2. What is the matrix representation of L_3 which is L_1 followed by L_2.

6. **[6+5+5]** Consider the three vectors $u = (-2, 1, 3)$, $v = (3, 0, 1)$ and $w = (0, 3, -7)$.

(a) Find the scalar triple $u \cdot (v \times w)$.

(b) What is the angle between u and v?

(c) What is the orthogonal projection of u onto v?

7. **[5+3+3+2]** Consider the vector space R^4 with the usual inner product. Let $S = \{v_1, v_2, v_3\} = \{(1,0,1,0), (0,1,0,0), (2,0,-2,0)\}$ be a subset in R^4.

(a) Show that S is an orthogonal set.

(b) Show using definition that S is linearly independent.

(c) What is the coordinate vector of $u = (1, 2, -3, 0)$ with respect to the orthogonal basis S.

(d) Orthonomalize the set S to get an orthonormal set R.

8. **[10]** Solve the system below using eigenvalues and eigenvectors of its coefficient matrix:

$$\begin{cases} y_1' = y_1 + y_2 \\ y_2' = 4y_1 - 2y_2 \end{cases}$$, with initial conditions $y_1(0) = 1,\ y_2(0) = 6$.

[If you need space write on back of this page]