Math 221

Major Exam 2

Problem 1: Let $f(x) = tanx$ and $x_0 = 0, x_1 = 0.6, x_2 = 0.9$. Construct interpolation polynomial of degree at most two to approximate $f(0.45)$, and find the absolute error.

Problem 2: Compute the linear least squares polynomial for the following data.

i	x_i	y_i
1	0	1
2	0.25	1.2840
3	0.50	1.6487
4	0.75	2.1170
5	1.00	2.7183

Problem 3: Find the least squares polynomial approximation to $f(x) = e^x$ on the interval $[0,2]$.

Problem 4: Use the forward-difference formula to approximate the derivative of $f(x) = lnx$ at $x_0 = 0.1$ using $h = .05$, and determine bounds for approximation error.

Problem 5: Approximate the following integrals using Trapezoidal rule.

a) $\int_0^{0.35} \frac{2}{x^2-4} dx$

b) $\int_0^1 x^2 e^{-x} dx$

Problem 6: Compute the eigenvalues and associated eigenvectors of the following matrix:

$$\begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}$$

Problem 7: Show that the following initial-value problem has a unique solution. $y' = y\cos(t)\,,\ 0 \le t \le 1,\ y(0) = 1.$