Prince Sultan University
Department of Mathematics and Physical Sciences

Math 221
Final Exam
Fall 2014
Wednesday, January 1, 2014

Time Allowed: 90 minutes

Name:
Student Number:

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard.
6. Looking around or making an attempt to cheat will result in your exam being cancelled.

Problem 1: (10 points) Use Newton's method to find a solution accurate to within 10^{-4} for $x - cosx = 0, \quad \left[0, \frac{\pi}{2}\right]$.

Problem 2: (10 points) Use fixed point iteration method to determine a solution accurate to within 10^{-2} for $x^3 - x - 1 = 0$ on $[1,2]$. Use $p_0 = 1$.

Problem 3: (10 points) Let $f(x) = cosx$ and $x_0 = 0, x_1 = 0.6, x_2 = 0.9$. Construct interpolation polynomial of degree at most two to approximate $f(0.45)$, and find the absolute error.

Problem 4: (10 points) Compute the linear least squares polynomial for the following data.

i	x_i	y_i
1	0	1
2	0.15	1.004
3	0.31	1.031
4	0.5	1.117
5	0.6	1.223
6	0.75	1.422

Problem 5: (10 points) Find the linear least squares polynomial approximation to $f(x) = \frac{1}{2}cosx + \frac{1}{3}sin2x$ on the interval $[0,1]$.

Problem 6: (10 points) Approximate the following integrals using Trapezoidal rule.

a) $\int_0^{\frac{\pi}{4}} x\sin x\, dx$

b) $\int_0^{\frac{\pi}{4}} e^{3x}\sin 2x\, dx$

Problem 7: (10 points) Compute the eigenvalues and associated eigenvectors of the following matrix:

$$\begin{pmatrix} 24 & 6 & 0 \\ 0 & 24 & 3 \\ 0 & 0 & 224 \end{pmatrix}$$

Problem 8: (10 points) Show that the following initial-value problem has a unique solution. $y' = -\frac{2}{t}y + t^2e^t,\ 1 \le t \le 2,\ y(1) = \sqrt{2}e.$