Prince Sultan University

Math 113
Major Exam 2
Second Semester, Term 162
Wednesday, May 3, 2017
Time Allowed: 90 minutes

Student Name:______________ Key ______________ __________________

Student ID #: __________________

Instructor: Dr. Jamiiru Luttamaguzi

Important Instructions:

1. You may NOT use notes or any textbook.
2. Talking during the examination is NOT allowed.
3. Your exam will be taken immediately if your mobile phone is seen or heard.
4. Looking around or making an attempt to cheat will result in your exam being cancelled.
5. This examination has **7** problems. Make sure your paper has all these problems.
6. Show your working.

Problems	Max points	Student's Points
Q#1	20	
Q#2,3	16	
Q#4,5	23	
Q#6,7	21	
Total	80	

Q#1 [20 Marks] Evaluate the integrals below

$$\int \frac{1+\sqrt{x}}{2x}dx = \int\left(\frac{1}{2x} + \frac{\sqrt{x}}{2x}\right)dx = \int\left(\frac{1}{2x} + \frac{x^{-\frac{1}{2}}}{2}\right)dx = \boxed{\frac{\ln|x|}{2} + \sqrt{x} + c}$$

$$\int te^{2t}dt = \frac{te^{2t}}{2} - \int \frac{e^{2t}}{2}dt$$

$u = t, \quad dv = e^{2t}dt$

$du = dt, \quad v = \frac{e^{2t}}{2}$

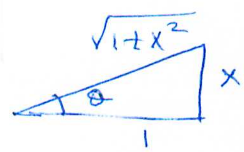
$$= \boxed{\frac{te^{2t}}{2} - \frac{e^{2t}}{4} + c}$$

$$\int \frac{x}{\left(1+x^2\right)^{3/2}}dx$$ (You must use trigonometric substitution)

$x = \tan\theta, \quad dx = \sec^2\theta\, d\theta$

$$= \int \frac{\tan\theta}{(1+\tan^2\theta)^{3/2}} \cdot \sec^2\theta\, d\theta = \int \frac{\tan\theta}{\sec^3\theta} \cdot \sec^2\theta\, d\theta$$

$$= \int \frac{\tan\theta}{\sec\theta}\, d\theta = \int \sin\theta\, d\theta = -\cos\theta + c$$



$$= \boxed{-\frac{1}{\sqrt{1+x^2}} + c}$$

$$\int_0^{\pi/8} \sin^2(2x)\cos^5(2x)dx = \int_0^{\pi/8} \sin^2(2x)\left(1-\sin^2(2x)\right)^2 \cdot \cos(2x)\,dx$$

$u = \sin(2x)$

$\frac{du}{2} = \cos(2x)\,dx$

x	u
0	0
$\frac{\pi}{8}$	$\frac{\sqrt{2}}{2}$

$$= \int_0^{\sqrt{2}/2} u^2(1-u^2)^2 \frac{du}{2}$$

$$= \frac{1}{2}\int_0^{\sqrt{2}/2} (u^2 - 2u^4 + u^6)\,du$$

$$= \frac{1}{2}\left[\frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7}\right]_0^{\sqrt{2}/2}$$

$$= \frac{1}{2}\left(\frac{1}{3}\left(\frac{\sqrt{2}}{2}\right)^3 - \frac{2}{5}\left(\frac{\sqrt{2}}{2}\right)^5 + \frac{1}{7}\left(\frac{\sqrt{2}}{2}\right)^7\right)$$

$$= \boxed{\frac{71\sqrt{2}}{3360} \simeq 0.03}$$

Q#2 [6 Marks] Write each rational expression as a partial fraction decomposition. **Do not find the constants.**

$\frac{x^2-5}{x^3-x} = \frac{x^2-5}{x(x-1)(x+1)}$ $= \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$	$\frac{7}{x(x-4)^3}$ $= \frac{A}{x} + \frac{B}{x-4} + \frac{C}{(x-4)^2} + \frac{D}{(x-4)^3}$	$\frac{3x^2-6x+2}{(x+7)(7x^2+2x+19)}$ $= \frac{A}{x+7} + \frac{Bx+C}{7x^2+2x+19}$

Q#3 [10 Marks] Determine whether each integral converges and find its limit, otherwise show that it diverges.

$$\int_0^\infty \frac{x}{x^2+8}dx = \lim_{x\to\infty} \int_0^x \frac{t}{t^2+8}dt = \lim_{x\to\infty} \frac{1}{2}\left[\ln(t^2+8)\right]_0^x$$

$$= \lim_{x\to\infty} \frac{1}{2}\left[\ln(x^2+8) - \ln 8\right]$$

$$= \boxed{\infty \quad \text{Diverges.}}$$

$$\int_0^1 \frac{3}{x^{1/5}}dx = \lim_{x\to 0^+} \int_x^1 \frac{3}{t^{1/5}}dt = \lim_{x\to 0^+} \int_x^1 3t^{-1/5}dt$$

$$= \lim_{x\to 0^+} \left[\frac{3t^{4/5}}{4/5}\right]_x^1 = \lim_{x\to 0^+} \frac{15}{4}\left[t^{4/5}\right]_x^1$$

$$= \lim_{x\to 0^+} \frac{15}{4}\left[1 - x^{4/5}\right]$$

$$= \frac{15}{4}[1-0] = \boxed{\frac{15}{4} \quad \text{Converges}}$$

Q#4 [20 Marks] Determine whether each sequence converges and find its limit, otherwise state that it diverges.

$\{a_n\} = \left\{1 + \frac{9^n}{10^n}\right\}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(1 + \left(\frac{9}{10}\right)^n\right) = 1 + 0 = \boxed{1}$$

$\{b_n\} = \left\{\frac{n}{\sqrt{3n^2+1}}\right\}$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n}{\sqrt{3n^2+1}} = \lim_{n\to\infty} \frac{\frac{n}{n}}{\sqrt{\frac{3n^2}{n^2} + \frac{1}{n^2}}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{3 + \frac{1}{n^2}}} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$$

$\{c_n\} = \left\{\frac{n}{e^n}\right\}$

$$\lim_{n\to\infty} c_n = \lim_{n\to\infty} \frac{n}{e^n} \overset{\text{LHopital's}}{\underset{\text{rule}}{=}} \lim_{n\to\infty} \frac{1}{e^n} = \boxed{0}$$

$\{d_n\} = \{\tan^{-1}(n)\}$

$$\lim_{n\to\infty} d_n = \lim_{n\to\infty} \tan^{-1} n = \boxed{\frac{\pi}{2}}$$

Q#5 [3 Marks] Find the n[th] partial sum of the series: $\sum_{n=0}^{\infty}\left(2\cdot\frac{4^{4n}}{3^{3n}}\right) = \sum_{n=0}^{\infty} 2\left(\frac{16}{27}\right)^n$

$$a = 2, \quad r = \frac{16}{27}.$$

$$S_n = \frac{a(1-r^{n+1})}{1-r} = \frac{2\left(1 - \left(\frac{16}{27}\right)^{n+1}\right)}{1 - \frac{16}{27}}$$

$$= \frac{2\left(1 - \left(\frac{16}{27}\right)^{n+1}\right)}{11/27}$$

$$= \boxed{\frac{54}{11}\left(1 - \left(\frac{16}{27}\right)^{n+1}\right)}$$

Q#6 [4 Marks] Consider the curve $x = y^2 - 2y$ on $0 \le y \le 2$. **Setup** integral for the surface area when the curve is rotated about the x-axis.

$$\frac{dx}{dy} = 2y - 2$$

$$A = 2\pi \int_c^d y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy = \boxed{2\pi \int_0^2 y \sqrt{1 + (2y-2)^2}\, dy}$$

Q#7 [17 Marks] Find the sum of the series if the series converges otherwise state that it diverges. Show your working.

$$\sum_{n=2}^{\infty} \left(\frac{6}{\sqrt{n}} - \frac{6}{\sqrt{n+1}} \right) = \lim_{n\to\infty} \sum_{k=2}^{n} \left(\frac{6}{\sqrt{k}} - \frac{6}{\sqrt{k+1}} \right)$$

$$= \lim_{n\to\infty} \left(\left(\frac{6}{\sqrt{2}} - \cancel{\frac{6}{\sqrt{3}}} \right) + \left(\cancel{\frac{6}{\sqrt{3}}} - \cancel{\frac{6}{\sqrt{4}}} \right) + \dots + \left(\cancel{\frac{6}{\sqrt{n}}} - \frac{6}{\sqrt{n+1}} \right) \right)$$

$$= \lim_{n\to\infty} \frac{6\sqrt{2}}{2} - \frac{6}{\sqrt{n+1}} = \boxed{3\sqrt{2}}$$

$$\frac{7}{9} - \frac{7}{27} + \frac{7}{81} - \frac{7}{243} + \cdots$$

$$a = \frac{7}{9}, \quad r = -\frac{1}{3}$$

$$S = \frac{a}{1-r} = \frac{7/9}{1 - {}^-1/3} = \frac{7/9}{4/3} = \boxed{\frac{7}{12}}$$

$$\sum_{n=0}^{\infty} 4^{4n} \cdot e^{-n} = \sum_{n=0}^{\infty} \left(\frac{16}{e}\right)^n, \quad a = 1, \quad r = \frac{16}{e} > 1$$

The series diverges.

$\sum_{n=1}^{\infty} a_n$ with partial sum $s_n = \dfrac{n}{4n+1}$

LHopital's rule

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n}{4n+1} = \lim_{n\to\infty} \frac{1}{4} = \frac{1}{4}$$

$$= \boxed{\frac{1}{4}}$$