Prince Sultan University
Deanship of Educational Services
Department of Mathematics
and General Sciences

COURSE DETAILS:

Calculus II	MATH 113	MAJOR EXAM I
Semester:	Spring Semester --Term 172	
Date:	Wednesday February 28, 2018	
Time Allowed:	90 minutes	

STUDENT DETAILS:

Student Name:	Key
Student ID Number:	
Section:	
Instructor's Name:	

INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

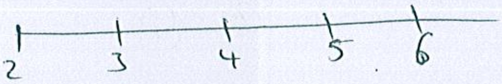
GRADING:

	Page 1	Page 2	Page 3	Page 4	Total	Total
Questions	1,2,3	4	5,6	7,8	8	
Marks	12	9	8	11	40	20

Q#1 [4 Marks] Estimate the area under the graph of $f(x)=\pi^x$ from $x=2$ to $x=6$ using four approximating rectangles and <u>midpoints</u>. Round your estimate to two decimal places.

<u>The solution:</u>

(1) $\Delta x = \frac{6-2}{4} = 1$



(1) $A \simeq f(2.5)+f(3.5)+f(4.5)+f(5.5)$

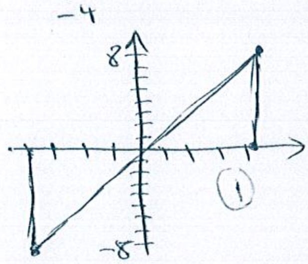
(1) $= \pi^{2.5}+\pi^{3.5}+\pi^{4.5}+\pi^{5.5}$

(1) $\simeq 787.51$

Q#2 [4 Marks] Evaluate the integral $\int_{-4}^{4}\left(2x+\sqrt{16-x^2}\right)dx$ by interpreting it as the area of a region. Sketch the region(s).

<u>The solution:</u>

$\int_{-4}^{4} 2x\,dx$



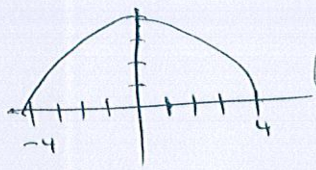
(1)

$A=0$ (1)

$\int_{-4}^{4}\sqrt{16-x^2}\,dx$

$y^2=16-x^2$

$\Rightarrow x^2+y^2=16$



(1)

$A=\frac{16\pi}{2}=8\pi$ (1)

$\Rightarrow \int_{-4}^{4}\left(2x+\sqrt{16-x^2}\right)dx = 8\pi$

Q#3 [4 Marks] On what interval is the function f below increasing?

$$f(x)=\int_{2}^{e^{x+1}} \ln t\,dt$$

<u>The solution:</u>

$f'(x)=\ln e^{x+1}\cdot e^{x+1} = (x+1)e^{x+1}$ (1) (1)

if $x+1>0 \Rightarrow x>-1$, then $f'(x)>0 \Rightarrow f$ is increasing (1)

$\Rightarrow f$ is increasing on $(-1,\infty)$ (1)

Q#4 [3 Marks Each] Evaluate the indefinite integrals:

1. $\int (8+\tan^2 x)\,dx \quad = I$

The solution:

$$I = \int [7 + (1+\tan^2 x)]\,dx \quad (1) = \int (7 + \sec^2 x)\,dx \quad (1)$$
$$= 7x + \tan x + C \quad (1)$$

2. $\int \frac{dx}{\sqrt{x} \cdot e^{4\sqrt{x}}} \quad = I$

The solution:

Let $u = \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}}\,dx$

$$\Rightarrow 2\int \frac{dx}{2\sqrt{x}\, e^{4\sqrt{x}}} = 2\int \frac{du}{e^{4u}} \quad (1) = 2\int e^{-4u}\,du = 2\,\frac{e^{-4u}}{-4} + C \quad (1)$$
$$= \frac{e^{-4\sqrt{x}}}{-2} + C \quad (1)$$

3. $\int \frac{\cos x}{\sqrt{1-\sin^2 x}}\,dx \quad = I$

The solution:

Let $u = \sin x \Rightarrow du = \cos x\,dx$

$$I = \int \frac{du}{\sqrt{1-u^2}} \quad (1) = \sin^{-1} u + C \quad (1)$$
$$= \sin^{-1}(\sin x) + C \quad (1)$$

Q#5 [3 Marks] Evaluate the definite integral: $\int_{e}^{e^2} \frac{(1+\ln x)^2}{x} dx$ $= I$

The solution:

Let $u = 1 + \ln x \Rightarrow du = \frac{1}{x} dx$

$x = e \Rightarrow u = 2$

$x = e^2 \Rightarrow u = 3$

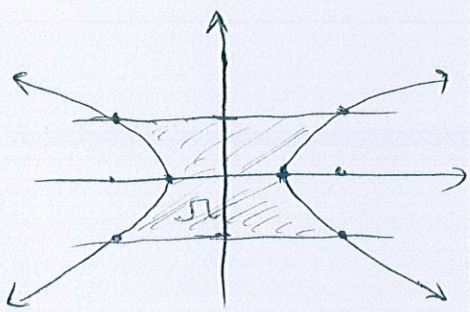
$$I = \int_2^3 u^2 \, du = \frac{u^3}{3}\Big|_2^3 = \frac{27-8}{3} = \frac{19}{3}$$

(1) (1) (1)

Q#6 [2+3 Marks] Let Ω denoted to the region bounded by $x = y^2 + 1$, $x = -y^2 - 1$, $y = 1$ and $y = -1$.

1. Sketch Ω.

The solution:



(2)

2. Find the area of Ω.

The solution:

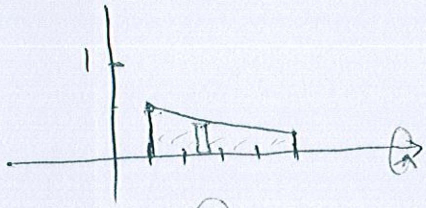
$$A = \int_{-1}^{1} [y^2 + 1 - (-y^2 - 1)] \, dy \quad (2)$$

$$= \int_{-1}^{1} (2y^2 + 2) \, dy = 2\left(\frac{y^3}{3} + y\right)\Big|_{-1}^{1}$$

$$= 2\left[\frac{4}{3} - \frac{-4}{3}\right] = \frac{16}{3} \quad (1)$$

Q#7 [5 Marks] Find the volume of the solid obtained by rotating the region bounded by $y = \frac{1}{x}$, $y = 0$, $x = 2$ and $x = 5$ about the x-axis. Sketch the region.

The solution:



$$V = \pi \int_2^5 \frac{1}{x^2}\, dx \quad (2)$$

$$= \pi \left(-\frac{1}{x}\right)_2^5 = \frac{3\pi}{10} \quad (1)$$

Q#8 [6 Marks] Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = -2$. Sketch the region.

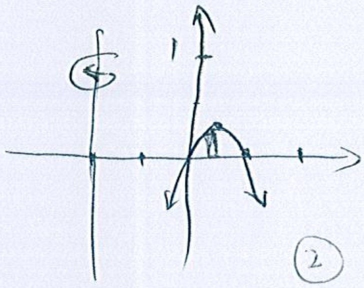
The solution:

$h = \frac{1}{2}$

$k = \frac{1}{4}$

$x(1-x) = 0$

$\Rightarrow 0, 1$



$$V = 2\pi \int_0^1 (x+2)(x-x^2)\, dx \quad (2)$$

$$= 2\pi \int_0^1 (x^2 - x^3 + 2x - 2x^2)\, dx = 2\pi \int_0^1 (-x^3 - x^2 + 2x)\, dx$$

$$= 2\pi \left[-\frac{x^4}{4} - \frac{x^3}{3} + 2\frac{x^2}{2}\right]_0^1 = 2\pi\left(-\frac{1}{4} - \frac{1}{3} + 1\right) = \frac{5\pi}{6} \quad (2)$$