Prince Sultan University

Math 113
Major Exam 1
Second Semester, Term 162
Sunday, March 19, 2017

Time Allowed: 75 minutes

Key

Student Name:________________ ____________________ ________________________

Student ID #: ____________________

Serial Class #:_____ Section #:

Circle your instructor's Name:

1. Dr. Jamiiru Luttamaguzi
2. Prof. Wasfi Shatanawi

Important Instructions:

1. You may NOT use notes or any textbook.
2. Talking during the examination is NOT allowed.
3. Your exam will be taken immediately if your mobile phone is seen or heard.
4. Looking around or making an attempt to cheat will result in your exam being cancelled.
5. This examination has **8** problems. Make sure your paper has all these problems.

Problems	Max points	Student's Points
Q#1,2	10	
Q#3,4,5	12	
Q#6,7	10	
Q#8	8	
Total	40	

Q#1 [6 Marks] Using the **Definition of the definite integral** (use the right end points) to evaluate

$$\int_0^2 (x^2-1)dx = \lim_{n\to\infty} \Delta x \sum_{i=1}^{n} f(a+i\Delta x)$$

$a=0,\ b=2,$
$\Delta x = \frac{2-0}{n} = \frac{2}{n}$

$$= \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n} f\left(\frac{2i}{n}\right) = \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n}\left(\left(\frac{2i}{n}\right)^2-1\right)$$

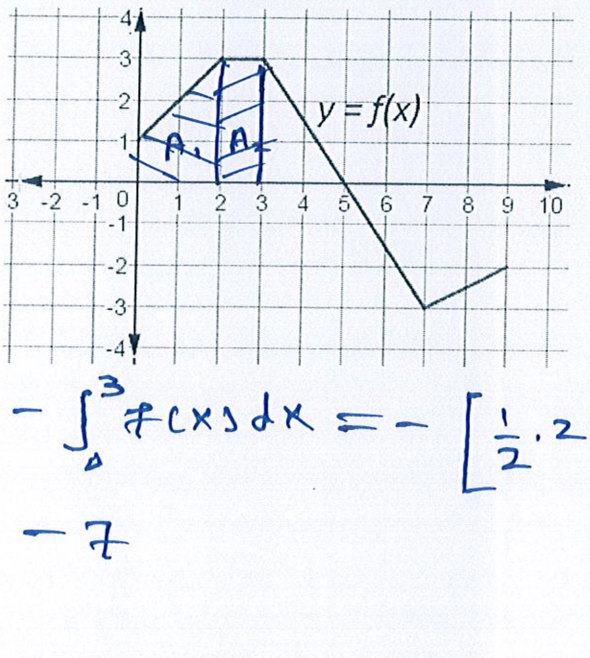
$$= \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n}\left(\frac{4i^2}{n^2}-1\right) = \lim_{n\to\infty} \frac{2}{n}\left[\frac{4}{n^2}\sum_{i=1}^{n} i^2 - \sum_{i=1}^{n} 1\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[\frac{4}{n^2}\cdot\frac{n(n+1)(2n+1)}{6} - n\right]$$

$$= \lim_{n\to\infty}\left[\frac{4}{3}\frac{(2n^2+3n+1)}{n^2} - 2\right]$$

$$= \lim_{n\to\infty}\left[\frac{4}{3}\left(2+\frac{3}{n}+\frac{1}{n^2}\right)-2\right] = \frac{8}{3}-2 = \frac{2}{3}$$

Q#2[4 Marks] Consider the function $f(x)$ graphed below



1. Evaluate $\int_3^0 f(x)\,dx$. $= -\int_0^3 f(x)dx = -\left[\frac{1}{2}\cdot 2(1+3) + 3\times 1\right]$

$= -7$

2. Find the average value of f on the interval [0, 3].

$$f_{ave} = \frac{1}{3-0}\int_0^3 f(x)dx \overset{\text{above}}{=} \frac{1}{3}\times 7 = \frac{7}{3}$$

Q# 3[4 Marks] Evaluate $\int \frac{1}{(1+t^2)\tan^{-1} t} dt$ $= \int \frac{1}{\tan^{-1} t} \cdot \frac{dt}{(1+t^2)}$

$u = \tan^{-1} t$

$du = \frac{dt}{1+t^2}$

$= \int \frac{du}{u} = \ln|u| + c$

$= \ln|\tan^{-1} t| + c$

Q#4.[4 Marks] Let $f(x) = (x-1)\int_4^{x^2} \frac{t^2}{\sqrt[8]{1+t^2}} dt$. Find

1. All value of ~~x~~ that makes $f(x) = 0$. $(x-1)\int_4^{x^2} \frac{t^2}{\sqrt[8]{1+t^2}} dt = 0$

when $x - 1 = 0$ or $x^2 = 4$

$x = 1$, or $x = -2$, $x = 2$

s.s. $= \{-2, 1, 2\}$

2. $f'(2) =$

$f'(x) = 1 \cdot \int_4^{x^2} \frac{t^2}{\sqrt[8]{1+t^2}} dt + (x-1) \cdot 2x \cdot \frac{x^4}{\sqrt[8]{1+x^4}}$

$f'(2) = \cancelto{0}{\int_4^4 \frac{t^2}{\sqrt[8]{1+t^2}} dt} + (2-1) \cdot 2(2) \cdot \frac{2^4}{\sqrt[8]{1+2^4}}$

$= \frac{64}{\sqrt[8]{17}}$

Q#5.[4 Marks] Evaluate $\int x(x+1)^5 \, dx$

$u = x + 1$

$x = u - 1$

$dx = du$

$\int x(x+1)^5 dx = \int (u-1) u^5 du$

$= \int u^6 - u^5 du = \frac{u^7}{7} - \frac{u^6}{6} + c$

$= \frac{(x+1)^7}{7} - \frac{(x+1)^6}{6} + c$

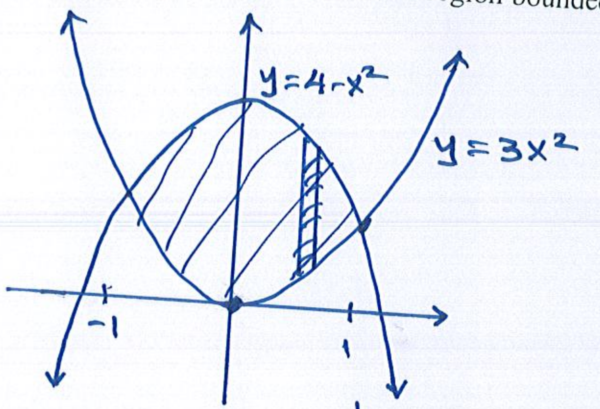
Q#6.[4 Marks] Evaluate $\int\left(\frac{2x^2+\sqrt[4]{x}-1}{x^2}\right)dx$ $= \int\left(\cancel{2}2 + \frac{x^{\frac{1}{4}}}{x^2} - \frac{1}{x^2}\right)dx$

$= \int\left(2 + x^{-\frac{7}{4}} - x^{-2}\right)dx$

$= 2x + \frac{x^{-3/4}}{-3/4} - \frac{x^{-1}}{-1} + c$

$= 2x \cancel{\pm} - \frac{4x^{-3/4}}{3} + x^{-1} + c = 2x \cancel{\pm} - \frac{4}{3x^{3/4}} + \frac{1}{x} + c$

Q#7[6 Marks] Find **the area** of the region bounded by the curves $y=3x^2$, $y=4-x^2$.



$3x^2 = 4 - x^2$

$4x^2 = 4$

$x^2 = 1$

$x = -1, 1.$

Area $= \int_a^b [f(x) - g(x)]\,dx = \int_{a=-1}^{b=1} [4 - x^2 - 3x^2]\,dx$

$= \int_{-1}^{1} (4 - 4x^2)\,dx = \left[4x - \frac{4x^3}{3}\right]_{-1}^{1}$

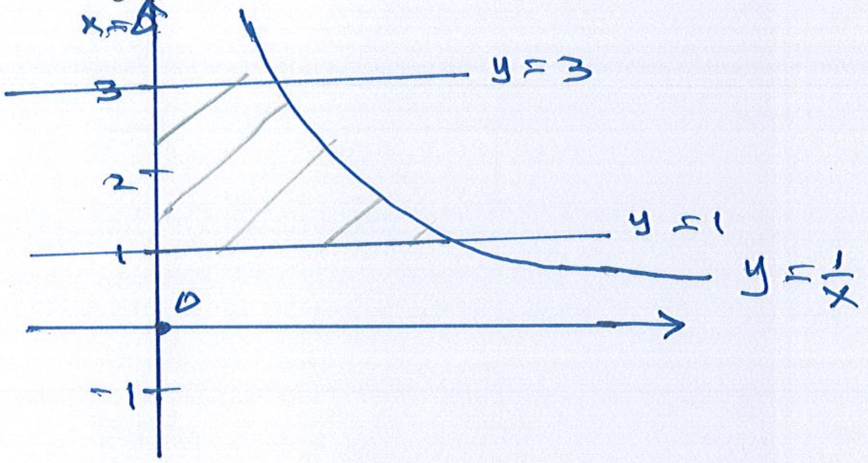
$= \left[\left(4 - \frac{4}{3}\right) - \left(-4 + \frac{4}{3}\right)\right]$

$= 4 - \frac{4}{3} + 4 - \frac{4}{3}$

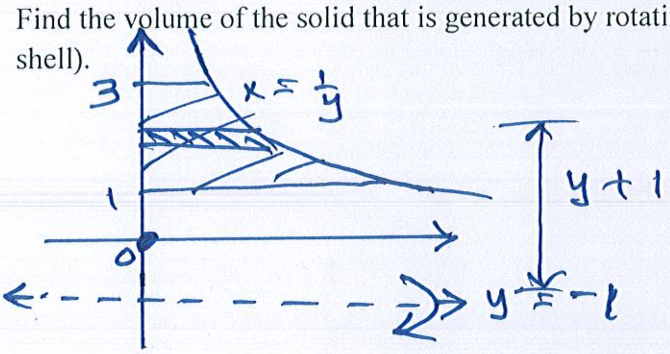
$= \frac{16}{3}$

Q#8[2+3+3 Marks] Let Ω denote to the region bounded by the curves $y=\frac{1}{x}, x=0,\ y=1$ and $y=3$.

1. Sketch the graph of Ω.



2. Find the volume of the solid that is generated by rotating Ω about the line $y=-1$. (Using cylindrical shell).



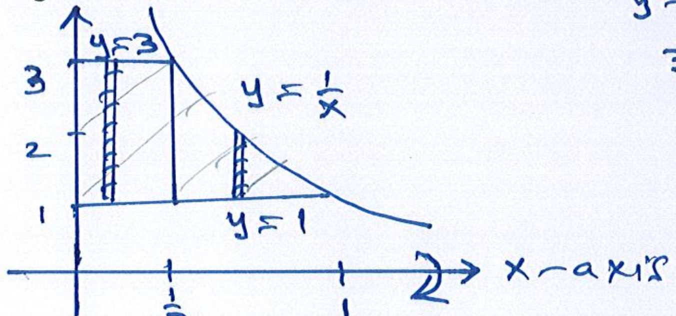
$$V_{shell} = 2\pi \int_1^3 (y+1)\cdot\frac{1}{y}\,dy$$

$$= 2\pi \int_1^3 \left(1+\frac{1}{y}\right)dy$$

$$V_{shell} = 2\pi\left[y+\ln y\right]_1^3 = 2\pi\left[(3+\ln 3)-(1+\ln 1)\right]$$

$$= 2\pi\left[2+\ln 3\right] = 4\pi + 2\pi\ln 3$$

3. Set up the integrals that represent the volume of the solid that is generated by revolving Ω about x-axis by using disc/washer method.



$y=\frac{1}{x}$

$3=\frac{1}{x} \Rightarrow x=\frac{1}{3}$

$1=\frac{1}{x} \Rightarrow x=1$

$$V_{washer} = \pi\int_0^{\frac{1}{3}} (3^2-1^2)\,dx + \pi\int_{\frac{1}{3}}^{1}\left(\frac{1}{x^2}-1^2\right)dx$$

$$= \pi\int_0^{1/3} 8\,dx + \pi\int_{1/3}^{1}\left(\frac{1}{x^2}-1\right)dx$$

$$= \dots = 4\pi.$$