Please make your own copy from the solutions key and back the original one to me.

Prince Sultan University

Math 113
Major Exam 1
Second Semester, Term 161
Thursday, November 3, 2016

Time Allowed: 75 minutes

The Solutions Key

Student Name: ______________ ______________ ______________

Student ID #: ______________

Serial Class #: _____ Section #:

Circle your instructor's Name:

1. Dr. Baha Abdullah 2. Dr. JamiiruLuttamaguzi 3. Prof. WasfiShatanawi

Important Instructions:

1. You may NOT use notes or any textbook.
2. Talking during the examination is NOT allowed.
3. Your exam will be taken immediately if your mobile phone is seen or heard.
4. Looking around or making an attempt to cheat will result in your exam being cancelled.
5. This examination has **9** problems. Make sure your paper has all these problems.

Problems	Max points	Student's Points
Q#1,2,3	13	
Q#4,5,6,7	14	
Q#8,9	13	
Total	40	

Q#1 [5 Marks]Using the **Definition of the definite integral** (use the right end points) to evaluate $\int_0^2 (2x+3)dx$

$f(x) = 2x+3$, $a=0$, $b=2$, $\Delta x = \frac{b-a}{n} = \frac{2}{n}$

1 Mark $\boxed{\int_0^2 f(x)\,dx = \lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x_i}$

$x_i = a + i\Delta x = 0 + \frac{2i}{n} = \frac{2i}{n}$

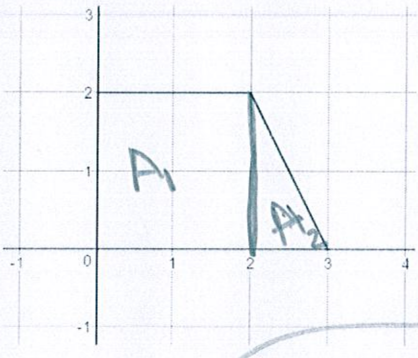
$f(x_i) = f\left(\frac{2i}{n}\right) = 2\left(\frac{2i}{n}\right) + 3 = \frac{4i}{n} + 3$ (1 Mark)

So $\sum_{i=1}^{n} f(x_i)\,\Delta x_i = \sum_{i=1}^{n}\left(\frac{4i}{n}+3\right)\frac{2}{n} = \boxed{\sum_{i=1}^{n}\frac{8i}{n^2} + \sum_{i=1}^{n}\frac{6}{n}}$

1 Mark $= \boxed{\frac{8}{n^2}\cdot\frac{n(n+1)}{2}} + \frac{6}{n}\cdot(n) = \frac{4(n+1)}{n} + 6$

1 Mark $\boxed{\text{So } \int_0^2 f(x)\,dx = \lim_{n\to+\infty}\left[\frac{4(n+1)}{n} + 6\right] = 4+6 = 10}$ (1 Mark)

Q#2[4 Marks] Consider the function $g(x)$ graphed below



1 Mark.

Evaluate the integral $\int_0^3 g(x)dx$. $= A_1 + A_2$

$= 2(2) + \frac{1}{2}(1)(2) = 4+1 = 5$

1 Mark 1 Mark 1 Mark,

Q# 3[4 Marks] Evaluate $\int \frac{dx}{\sqrt{1-x^2}\,\sin^{-1}x}$

1 Mark.

put $\boxed{u = \sin^{-1}x \Rightarrow du = \frac{1}{\sqrt{1-x^2}}\,dx \Rightarrow dx = \sqrt{1-x^2}\,du}$

$\int \frac{dx}{\sqrt{1-x^2}\,\sin^{-1}x} = \int \frac{\sqrt{1-x^2}\,du}{\sqrt{1-x^2}\,u} = \int \frac{du}{u} = \ln|u| + c$

$= \ln|\sin^{-1}x| + c$

2 marks. 1 Mark.

Q#4.[4 Marks] Let $f(x) = x^2 \int_1^{x^3} \frac{\sin t}{1+t^2}\, dt$. Find $f'(-1)$

2 Marks

$$f'(x) = x^2 \cdot \frac{\sin(x^3)}{1+x^6} \cdot 3x^2 + 2x \int_1^{x^3} \frac{\sin t}{1+t^2}\, dt$$

$$f'(x) = \frac{3\cancel{x}^4 \cdot \sin(x^3)}{1+x^6} + 2x \int_1^{x^3} \frac{\sin t}{1+t^2}\, dt$$

odd function

$$f'(-1) = \frac{3\sin(-1)}{2} + -2 \int_1^{-1} \frac{\sin t}{1+t^2}\, dt$$

1 Marks

$$= \frac{3}{2}\sin(-1) + -2(0) = \frac{3}{2}\sin(-1) = -1.262$$

1 Marks

Q#5.[3 Marks] Evaluate $\int \sqrt{e^x}\, dx$

$$\int \sqrt{e^x}\, dx = \int (e^x)^{\frac{1}{2}}\, dx = \int e^{\frac{1}{2}x}\, dx = 2e^{\frac{1}{2}x} + C$$

1 Marks   1 Marks   1 Marks

Q#6.[3 Marks] Evaluate $\int (x^2 + \sqrt[3]{x} + 3)\, dx$

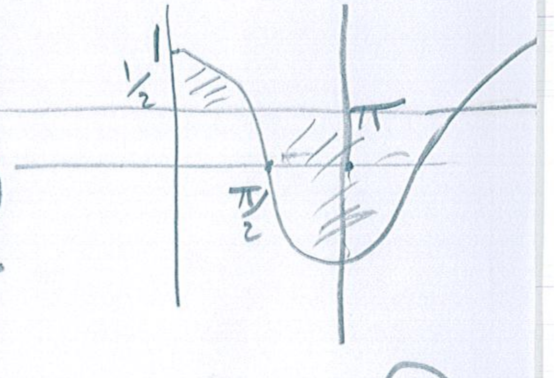
$$= \int \left(x^2 + x^{\frac{1}{3}} + 3\right) dx = \frac{1}{3}x^3 + \frac{3}{4}x^{\frac{4}{3}} + 3x + C$$

1 Marks   1 Marks   1 Marks

− 1 Mark if C is missing

Q#7[4 Marks]Find **the area** of the region bounded by the curves $y = \cos x$, $y = \frac{1}{2}$, and $x \in [0, \pi]$.

$$\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}$$



$$A = \int_0^{\pi/3} \left(\cos x - \frac{1}{2}\right) dx + \int_{\pi/3}^{\pi} \left(\frac{1}{2} - \cos x\right) dx$$

1 Marks   1 Marks

$$= \left(\sin x - \frac{1}{2}x\right)\Big|_0^{\frac{\pi}{3}} + \left(\frac{1}{2}x - \sin x\right)\Big|_{\pi/3}^{\pi}$$

1 Marks   1 Marks

$$= \left[\sin\left(\frac{\pi}{3}\right) - \frac{\pi}{6} - 0\right] + \left[\frac{\pi}{2} - \sin\pi - \left(\frac{\pi}{6} - \sin\left(\frac{\pi}{3}\right)\right)\right]$$

$$= \frac{\sqrt{3}}{2} - \frac{\pi}{6} + \frac{\pi}{2} - \frac{\pi}{6} + \frac{\sqrt{3}}{2} = \frac{\pi}{2} - \frac{2\pi}{6} + \sqrt{3} = \frac{\pi}{6} + \sqrt{3}$$

$= 2.2551$

1 mark

Q#8 [5 Marks] Find the **average value** of the function $f(x) = x\sqrt{x+1}$ on $[0,4]$.

1 marks

$$f_{ave} = \frac{1}{4}\int_0^4 f(x)\,dx$$

put $u = x+1$, $du = dx$

$$\int x\sqrt{x+1}\,dx = \int x\sqrt{u}\,du$$

1 marks

$$= \int (u-1)\sqrt{u}\,du$$

$$= \int u^{\frac{3}{2}} - u^{\frac{1}{2}}\,du$$

$$= \frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + c$$

$$= \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + c$$

1 marks.

$$= \frac{1}{4}\left[\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}}\right]_0^4$$

or $\frac{1}{4}\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right]_1^5$

$$= \frac{1}{4}\left[\frac{2}{5}(5)^{\frac{5}{2}} - \frac{2}{3}(5)^{\frac{3}{2}}\right] - \frac{1}{4}\left[\frac{2}{5} - \frac{2}{3}\right]$$

$$= \frac{2}{20}(5)^{\frac{5}{2}} - \frac{2}{12}(5)^{\frac{3}{2}} + \frac{1}{15}$$

$= 3.79$

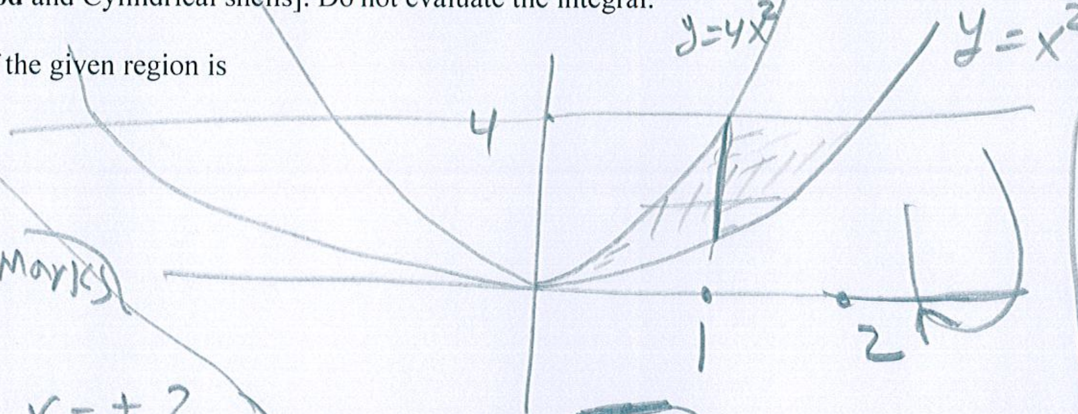
$= \frac{91}{24}$

1 Marks

Q#10[2+3+3 Marks] Graph the region bounded by the curves $y = 4x^2$, $y = x^2$, $y = 4$, and $x > 0$ **Then set up the integral** to find the volume of the solid generated by rotating the given about x-axis [**Using the Disk/Washer method** and Cylindrical shells]. Do not evaluate the integral.

The Solution:

1. The graph of the given region is



1 Marks

$4x^2 = 4$

$x = \pm 1$

1 Marks

$x^2 = 4 \Rightarrow x = \pm 2$

2. The integral to find the volume by using Disk and Washer is:

1 marks

$$V = \int_0^1 \left(\pi(16x^4) - \pi x^4\right)dx + \int_1^2 \left[\pi(16) - \pi(x^4)\right]dx$$

1 marks. 1 marks

1 Marks

3. The integral to find the volume by using cylindrical shells is:

$$V = \int_0^4 2\pi y\left(\sqrt{y} - \frac{1}{2}\sqrt{y}\right)dy = \int_0^4 \pi y\sqrt{y}\,dy = \int_0^4 \pi y^{\frac{3}{2}}\,dy$$

2 marks

1 marks $\frac{1}{2}$

$\frac{1}{2}$ marks