Prince Sultan University

Math 113
Major Exam 1
Second Semester, Term 152
Thursday, February 25, 2016

Time Allowed: 90 minutes

Student Name:_______________ __________________ _____________________

Student ID #: __________________ Key

Serial Class #:_____ Section #:

Circle your instructor's Name: 1. Dr. Aiman Mukheimer 2. Dr. Jamiiru Luttamaguzi

3. Prof. Wasfi Shatanawi

Important Instructions:

1. You may NOT use notes or any textbook.
2. Talking during the examination is NOT allowed.
3. Your exam will be taken immediately if your mobile phone is seen or heard.
4. Looking around or making an attempt to cheat will result in your exam being cancelled.
5. This examination has 7 problems, some with several parts. Make sure your paper has all these problems.

Problems	Max points	Student's Points
Q#1,2,3	18	
Q#4,5	12	
Q#6	10	
Q#7,8	14	
Q#9,10	26	
Total	80	

Q#1 (6 points) Using the **Definition of the integral** (use the right end points) to evaluate $\int_2^4 (2x - x^2)\, dx$

$f(x) = 2x - x^2, \quad a = 2, \quad b = 4, \quad \Delta x = \frac{b-a}{n} = \frac{4-2}{n} = \frac{2}{n}$

$\int_2^4 (2x - x^2)dx = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x\, f(a + i\Delta x) = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} f(2 + \frac{2i}{n})$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left[2(2+\frac{2i}{n}) - (2 + \frac{2i}{n})^2\right]$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left[2(\cancel{2} + \frac{2i}{n}) - (\cancel{4} + \frac{8i}{n} + \frac{4i^2}{n^2})\right]$

$= \lim_{n\to\infty} \sum_{i=1}^{n} -\frac{8i}{n^2} - \frac{8i^2}{n^3} = \lim_{n\to\infty}\left[-\frac{8}{n^3}\sum_{i=1}^{n} i^2 - \frac{8}{n^2}\sum_{i=1}^{n} i\right]$

$= \lim_{n\to\infty}\left[\frac{-8}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{8}{n^2}\frac{n(n+1)}{2}\right]$

$= \lim_{n\to\infty}\left[-\frac{4}{3}(1+\frac{1}{n})(2+\frac{1}{n}) - 4(1+\frac{1}{n})\right] = -\frac{4}{3}(2) - 4 = -\frac{20}{3}$

Q#2. (6 points) If $F(x) = \int_{4x^2}^{16} \frac{e^t}{t} dx$, find $F(2) + F'(2) + F''(2)$.

$F(2) = \int_{16}^{16} \frac{e^t}{t} dt = 0$

$F'(x) = 8x\left(-\frac{e^{4x^2}}{4x^2}\right) = -\frac{2e^{4x^2}}{x}; \quad F'(2) = -\frac{2e^{16}}{2} = -e^{16}$

$F''(x) = \frac{x(-16x e^{4x^2}) + 2e^{4x^2}}{x^2} = \frac{2e^{4x^2} - 16x^2 e^{4x^2}}{x^2}$

$F''(2) = \frac{2e^{16} - 64e^{16}}{4} = -\frac{31}{2}e^{16}$

$\therefore \quad F(2) + F'(2) + F''(2) = 0 - e^{16} - \frac{31}{2}e^{16} = -\frac{33}{2}e^{16}$

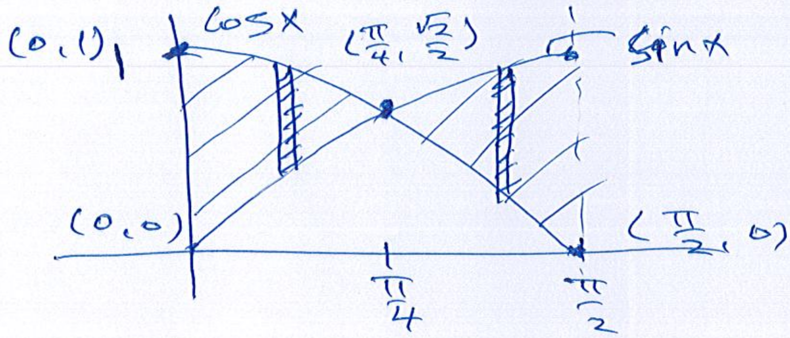
Q#3 (6 points) Evaluate $\int \frac{x^2+1}{2} + \frac{2}{1+x^2} dx \quad = \int \frac{x^2}{2} + \frac{1}{2} + \frac{2}{1+x^2}\, dx$

$= \frac{x^3}{6} + \frac{x}{2} + 2\tan^{-1}x + C$

Q#4 (2+6 points) Let Ω be the region bounded by the curves $y = \cos x$, $y = \sin x$, $x = 0$ and $x = \frac{\pi}{2}$.

1. **Sketch Ω.** (Your graph should contains all **intersection points, x-intercepts,** and **y-intercepts**)

$$\sin x = \cos x \Rightarrow \frac{\sin x}{\cos x} = 1 \Rightarrow \tan x = 1$$
$$x = \frac{\pi}{4}$$



2. Find area of Ω.

$$A = \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx$$
$$= \left[\sin x + \cos x\right]_0^{\pi/4} + \left[-\cos x - \sin x\right]_{\pi/4}^{\pi/2}$$
$$= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - (0+1) + (0-1) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)$$
$$= 2\sqrt{2} - 2$$

Q#5(4 points) **Evaluate** $\int e^{2x} + x^2 dx$

$$= \int e^{2x} dx + \int x^2 dx \quad ; \quad u = 2x$$
$$du = 2dx$$
$$= \int \frac{1}{2} e^u du + \frac{x^3}{3} + C \qquad \frac{du}{2} = dx$$
$$= \frac{1}{2} e^u + \frac{x^3}{3} + C$$
$$= \frac{1}{2} e^{2x} + \frac{x^3}{3} + C$$

Q#6 (2+4+4 points) Let Ω be the region bounded by the curves $y=\frac{1}{4}x^2$ and $y=\frac{1}{2}x+2$.

1. **Sketch Ω.** (Your graph should contains all **intersection points, x-intercepts**, and **y-intercepts**)

$\frac{1}{4}x^2 = \frac{1}{2}x + 2$

$x^2 = 2x + 8$

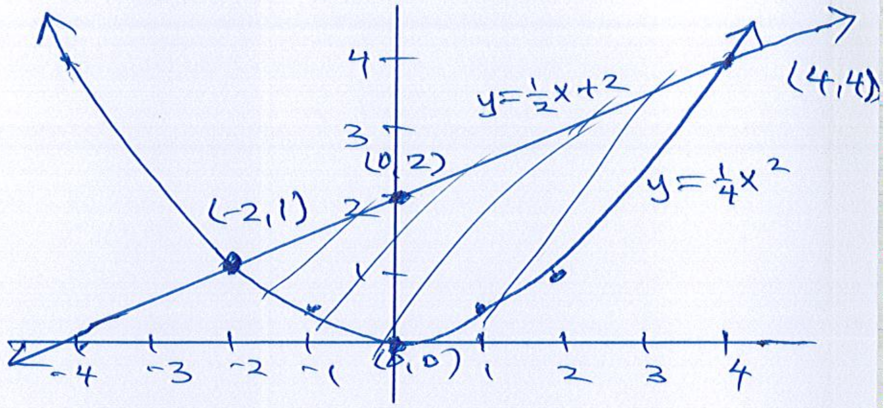
$x^2 - 2x - 8 = 0$

$(x+2)(x-4) = 0$

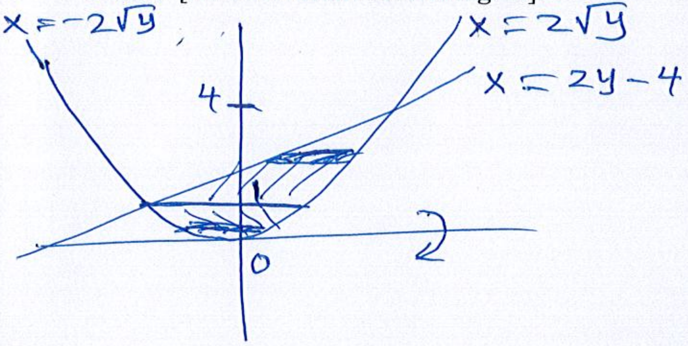
$x = -2 \quad x = 4$

$y = 1 \quad y = 4$

$(-2,1) \quad (4,4)$

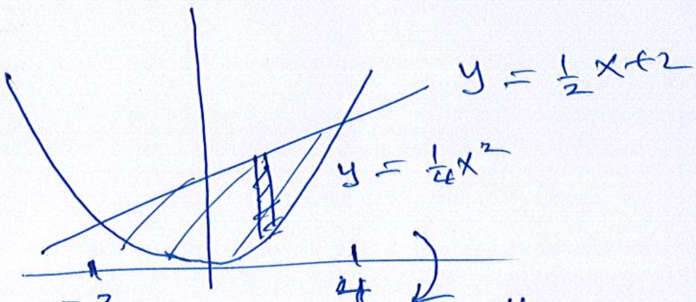


2. Use **cylindrical shell to setup** the integral for the volume of the solid generated by rotating Ω about the x-axis. **[Don't evaluate for integral]**.



$$V = 2\pi \int_0^1 y[2\sqrt{y} - -2\sqrt{y}]\,dy + 2\pi \int_1^4 y[2\sqrt{y} - 2y + 4]\,dy$$

3. Use **the disc and washer method to setup** the integral for the volume of the solid generated by rotating Ω about the x-axis. **[Don't evaluate for integral]**.



$$V = \pi \int_{-2}^{4} \left(\tfrac{1}{2}x+2\right)^2 - \left(\tfrac{1}{4}x^2\right)^2 dx.$$

Q#7 (6 points) Let f be an **even continuous** function and g be an **odd continuous** function on [-4,4]. Suppose that $\int_0^4 f(x)dx = 4$ and $\int_0^4 g(x)\,dx = 3$. **Find**

1. $\int_2^0 f(2x)dx$

$u = 2x$, $du = 2dx$, $\frac{du}{2} = dx$

x	u
0	0
2	4

$$= \int_4^0 \frac{1}{2} f(u)du$$

$$= -\frac{1}{2}\int_0^4 f(u)du$$

$$= -\frac{1}{2}(4) = \boxed{-2}$$

2. Find the average value of the function $1 + 2f + 3g$ on [-4,4].

$$\text{Average} = \frac{1}{4--4}\int_{-4}^{4} 1 + 2f(x) + 3g(x)\,dx$$

$$= \frac{1}{8}\left[\int_{-4}^{4} dx + 2\int_{-4}^{4} f(x)dx + 3\int_{-4}^{4} g(x)dx\right]$$

$$\text{Average} = \frac{1}{8}\left[[x]_{-4}^{4} + 4\int_0^4 f(x)dx + 3(0)\right]$$

$$= \frac{1}{8}[4+4+4\times 4] = \boxed{3}$$

Q#8(8 Marks) Find all number(s) of b such that the **average value** of $f(x) = 2 + 6x - 3x^2$ on the interval $[0,b]$ is equal to **3**.

$$f_{ave} = \frac{1}{b-0}\int_0^b f(x) = \frac{1}{b}\int_0^b [2+6x-3x^2]dx = 3$$

$$\frac{1}{b}\left[2x + 3x^2 - x^3\right]_0^b = 3$$

$$\frac{1}{b}(2b + 3b^2 - b^3) = 3$$

$$2 + 3b - b^2 - 3 = 0$$

$$b^2 - 3b + 1 = 0$$

$$b = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2} > 0$$

$$\therefore \boxed{b = \frac{3+\sqrt{5}}{2},\quad b = \frac{3-\sqrt{5}}{2}}$$

Q#9 (18 points) Evaluate the following integrals: **(Show your work in details)**

i. $\int_0^{\pi/4} \frac{\cos^3 x - 5}{\cos^2 x} dx = \int_0^{\pi/4} \frac{\cos^3 x}{\cos^2 x} - \frac{5}{\cos^2 x} dx = \int_0^{\pi/4} \cos x - 5\sec^2 x \, dx$

$= \int_0^{\pi/4} \cos x - 5\int_0^{\pi/4} \sec^2 x \, dx = [\sin x]_0^{\pi/4} - [5\tan x]_0^{\pi/4}$

$= \left(\frac{\sqrt{2}}{2} - 0\right) - (5 - 0) = \frac{\sqrt{2}}{2} - 5$

ii. $\int_e^{e^8} \frac{1}{x\sqrt[3]{\ln x}} dx \qquad u = \ln x \Rightarrow du = \frac{1}{x} dx$

x	u
e	1
e^8	8

$= \int_1^8 \frac{1}{\cancel{x}\sqrt[3]{u}} \cancel{x\,du} = \int_1^8 u^{-\frac{1}{3}} du = \left[\frac{3}{2} u^{\frac{2}{3}}\right]_1^8 = 6 - \frac{3}{2} = \frac{9}{2}$

iii. $\int_0^3 x|2 - x|dx \qquad |2 - x| = \begin{cases} x - 2, & 2 \le x \le 3 \\ 2 - x & 0 \le x \le 2 \end{cases}$ 2

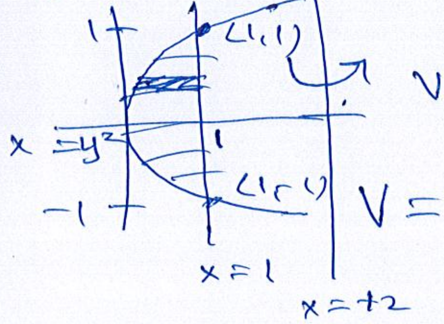
$= \int_0^2 x(2 - x)dx + \int_2^3 x(x - 2)dx = \int_0^2 2x - x^2 dx + \int_2^3 x^2 - 2x \, dx$ 2

$= \left[x^2 - \frac{x^3}{3}\right]_0^2 + \left[\frac{x^3}{3} - x^2\right]_2^3 = \left(4 - \frac{8}{3}\right) - 0 + \cancel{(9 - 9)} - \left(\frac{8}{3} - 4\right)$

$= 8/3$ 2

Q#10 (8 points) Find the **volume** of the solid that is generated by rotating the region bounded by $x = y^2$ and $x = 1$ about the line $x = 2$.

(1) Washer method



$V = \pi \int_{-1}^{1} f(y)^2 - g^2(y) dy$ 2

$V = \pi \int_{-1}^{1} (2 - y^2)^2 - 1^2 dy$ 2

$V = \pi \int_{-1}^{1} y^4 - 4y^2 + 3 dy$

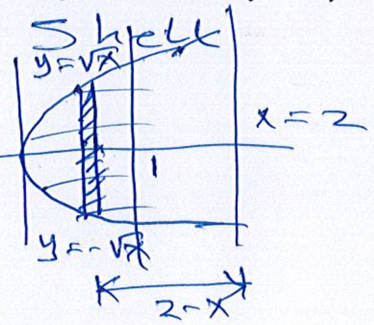
$= \pi \left[\frac{y^5}{5} - \frac{4y^3}{3} + 3y\right]_{-1}^{1}$ 2

$= \pi\left[\frac{1}{5} - \frac{4}{3} + 3\right] - \pi\left[-\frac{1}{5} + \frac{4}{3} - 3\right]$

$= \pi\left[\frac{2}{5} - \frac{8}{3} + 6\right]$

$= \frac{56\pi}{15}$ 2

(2) Shell



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$V = 2\pi \int_0^1 (2 - x)(\sqrt{x} - -\sqrt{x})dx$ 2

$= 2\pi \int_0^1 2\sqrt{x}(2 - x)dx$

$= 2\pi \int_0^1 4x^{\frac{1}{2}} - 2x^{\frac{3}{2}} dx$ 2

$= 2\pi \left[\frac{8}{3} x^{3/2} - \frac{4}{5} x^{5/2}\right]_0^1$ 1

$= 2\pi \left[\frac{8}{3} - \frac{4}{5}\right]$ 2

$= 2\pi \cdot \frac{28}{15} = \frac{56\pi}{15}$