



Prince Sultan University
Department of General Sciences
MATH 111
Final Exam
Semester I, Term 171
Saturday, December 30th, 2017

Time Allowed: **3 hours**

Student Name: _____

Student ID #: _____

Section #: _____

Teacher's Name: _____

Serial Class Number: _____

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 14 problems, some with several parts.

Problems	Max points	Student's Points
1,2,3	13	
4,5,6	15	
7,8,9	12	
10	20	
11	13	
12	15	
13,14	12	
Total	100	_____ % = _____/40

Q.1 (4 points) Let $f(x) = \begin{cases} |x+1| & x < -2 \\ x+1 & -2 \leq x < 1 \\ \sqrt{x+3} & 1 \leq x \leq 6 \end{cases}$. Find the values of x , if any, at which f is not continuous.

Show all your steps

Q2 (5 points) Find the equation of the tangent line to the curve of $y = \frac{x^2 + 1}{2 + \sin x}$ at $x = 0$

Q.3 (4 points) Let $H(x) = f(x \cdot g(x) - 2)$, if $f(4) = 6$, $f'(4) = -5$, $g(2) = 3$ and $g'(2) = -2$, find $H'(2)$.

Q.4 (5 points) Consider the curve $4x^2 + y^2 - 8x + 4y + 4 = 0$, **find all the points** where the curve has horizontal tangent lines, if any.

Q.5 (5 points) Let $e^{xy} = y^3 - x$, find the slope of the normal line at $x = 0$.

Q.6 (5 points) Let $f(x) = (x-1)^2(x-2)^2(x-3)^2$, find $\frac{f'(0)}{f(0)}$

Q.7 (4 points) **Find** $f'(1)$ if $f(x) = \frac{x^5 \cdot \sqrt[3]{3x-2}}{e^{x^2} (x^3+1)^{10}}$, [Hint: you may use logarithmic differentiation]

Q.8 (3 points) Let $f(x) = 5 + 6\sin(3x)$, find $f^{(87)}(x)$.

Q.9 (5 points) Let $f(x) = x^2 e^{-x}$, use the **Second Derivative Test** to find all the x -values of the local maximum and minimum (if any) for f .

Q.10 (20 points) Find the limit

Show your work in details

(a) $\lim_{x \rightarrow 0} \sqrt{\frac{\sin(2x) - x}{4x}}$

(b) $\lim_{x \rightarrow 0} \frac{7x^2}{1 - \cos^2 x}$

(c) $\lim_{x \rightarrow \infty} \left(2x \cdot \tan\left(\frac{8}{x}\right) \right)$

(d) $\lim_{x \rightarrow 0^+} \left[\frac{1}{x} - \frac{1}{e^x - 1} \right]$

(e) $\lim_{x \rightarrow \infty} (e^x + 1)^{\frac{-2}{x}}$

Q.11 (13 points) Find the derivative of each problem below **without simplifying.**

(a) (2 points) $y = \tan^{-1}(\cot(x))$

(b) (3 points) $y = x^3 \ln(\cos(x^3)) + \cos(\ln(x^2))$

(c) (2 points) $y = \frac{x^2 - 1}{2x + e^{3x}}$

(d) (3 points) $y = (\tan x)^{\cos^{-1}(x)}$

(e) (3 points) $y = \frac{1}{3} \sinh^3(e^{7x}) + \sqrt[3]{\operatorname{sech}(x)}$

Q.12) (15 points) Let $f(x) = \frac{6x}{x^2 + 1}$.

Show your work in details

(a) (2 points) Find the domain of f and the y -intercept.

(b) (2 points) Determine the vertical and horizontal asymptotes, if any.(Show your work in details)

(c) (5 points) Find the critical numbers and the local maximum and/or local minimum points, if any.

(d) (3 points) Find the intervals on which f is concave up and/or concave down and the inflection point(s) if any.

(e) (3 points) Sketch the graph of f showing on the graph all significant features

Q.13 (7 points) Find the radius and the height that produce the **least amount of material** needed to construct a right circular can that will hold a volume of 1000 cm^3 . (Surface area of the cylinder is $2\pi r^2 + 2\pi rh$).

Q.14 (5 points) Find the **maximum vertical distance** between $y = x^2 - 1$ and $y = 2x + 7$ over $[-1, 3]$.