Prince Sultan University
Department of General Sciences
MATH 111
Final Exam
Semester I, Term 161
Tuesday, January 10th, 2017

Time Allowed: **3 hours**

Student Name: _____

Student ID #: _____   Section #: _____

Teacher's Name: _____

Serial Class Number: _____

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 14 problems, some with several parts.

Problems	Max points	Student's Points
1,2,3,4	15	
5	14	
6	18	
7,8	10	
9,10	13	
11	16	
12,13,14	14	
Total	**100**	_____% = _____/40

1. (2 points(CLO1)) The graph of a function g is shown in the figure. Use it to state the values (if they exist) of the following:

(a) $\lim\limits_{x \to 2^-} g(x) =$

(b) $\lim\limits_{x \to 2^+} g(x) =$

(c) $\lim\limits_{x \to 2} g(x) =$

(d) $\lim\limits_{x \to 5} g(x) =$
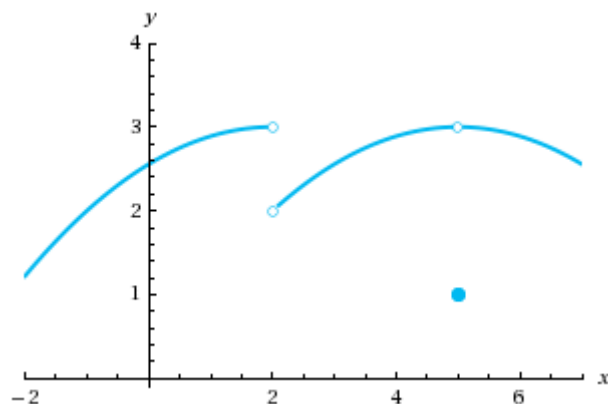
2. (5 points(CLO2)) Find equations of the tangent lines to the curve $y = \dfrac{x-1}{x+1}$ that are parallel to the line $x - 2y = 2$.

3. (5 points(CLO2)) Find equation(s) of the normal line(s) to the curve $x^2 + 2xy - y^2 + x = 2$ at $x = 1$.

4. (3 points(CLO2)) Suppose $f(\pi/3) = 4$, $f'(\pi/3) = -2$ and $h(x) = \dfrac{\cos x}{f(x)}$, Find $h'(\pi/3)$.

5. (14 points(CLO1)) Evaluate the limit, if it exists. You may use L'Hospital's rule (show your work in details).

(a) (3 points) $\lim_{x\to\infty} \sqrt[3]{\dfrac{2+3x-5x^2}{1+8x^2}}$

(b) (4 points) $\lim_{x\to 0^+} (\cos x)^{\frac{1}{x^2}}$

(c) (3 points) $\lim_{x\to 1^+} (\dfrac{1}{\ln x} - \dfrac{1}{x-1})$

(d) (4 points) $\lim_{x\to\infty} (e^x + x)^{\frac{1}{x}}$

6. (18 points(CLO2)) Find the derivative y' of each problem below simplifying where possible.

(a) (4 points) $x \cdot y = \sin^{-1} x - \sin y$ (Hint: Use Implicit differentiation)

(b) (4 points) $y = (3x^2 + 2x - 1)^{\sin x}$

(c) (3 points) $y = \sin^4(x^2 + 1)$

(d) (3 points) $y = 2^x - 3\tanh x - 4x$

(e) (4 points) $y = \dfrac{(x+5)^{10} \cosh^9 x \, \tan^3 x}{\sqrt[3]{x^2 + 7} \, \sqrt{x^4 + 7}}$ (Hint: Use logarithmic differentiation)

7. (5 points(CLO2)) Given is the function $g(x) = e^{x^3 - 3x}$
   (a) (2 points) Find the derivative $g'(x)$

   (b) (3 points) Find the absolute minimum and maximum of $g(x)$ on the interval [-2, 2]

8. (5 points(CLO3)) a) (2 points) Locate the discontinuities of the function. (show your work in details)

$$y = \frac{e^{\frac{4}{x}}}{6x^2 + x - 1}$$

b) (3 points) For what value of the constant c is the function f continuous on $(-\infty, \infty)$

$$f(x) = \begin{cases} cx^2 - 9x & \text{if } x \le 4 \\ \dfrac{4\sin(x-4)}{(x-4)} & \text{if } x > 4 \end{cases}$$

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9. (6 points(CLO4)) Let $f(x) = \tan^{-1}(\dfrac{x+1}{x-1})$.

(a) (1 point) Find the domain of $f(x)$

(b) (2 points) Compute $\lim\limits_{x \to \infty} f(x)$ and $\lim\limits_{x \to -\infty} f(x)$ if possible and give the equations of the vertical and horizontal asymptote(s), if any.

(c) (3 points) Compute the first derivative and determine the intervals in which $f(x)$ is increasing or decreasing.

10. (7 points(CLO5)) Let $f(x) = \dfrac{1}{x^2 - 64}$

(a) (1 point) Find the domain of $f(x)$

(b) (3 points) Compute $\lim\limits_{x \to \infty} f(x)$ and $\lim\limits_{x \to -\infty} f(x)$ if possible and give the equations of the vertical and horizontal asymptote(s), if any.

(c) (3 points) Find all local minima and maxima, if any.

11. (16 points(CLO5)) Let $f(x) = (x-3)\sqrt{x}$

(a) (1 point) Find the domain of $f(x)$

(b) (2 points) Find the y-intercept and the x-intercept(s) of $f(x)$.

(c) (1 point) Compute $\lim_{x \to \infty} f(x)$

(d) (1 point) Does the function $f(x)$ have special properties (is it even/odd/neither?)

(e) (2 points) Compute the first and the second derivative of $f(x)$.

(f) (3 points) Find all local minima and maxima, **and** determine the absolute maximum and minimum, if any.

(g) (2 points) Find the inflection point(s) of $f(x)$, if any.

(h) (4 points) Sketch the graph of f showing on the graph all significant features.
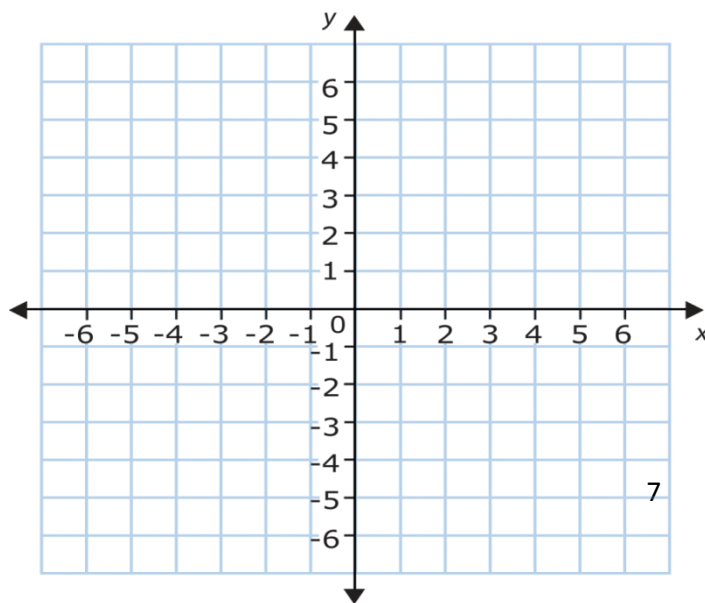
12. (5 points(CLO6)) Two cars start moving from the same point. One travels south at 48 km/h and the other travels west at 20 km/h. At what rate is the distance between the cars increasing <u>four</u> hours later?

13. (4 points(CLO6)) Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3,0)$

14. (5 points(CLO6)) A **<u>closed box</u>** with a square base must have a volume of 8,000 cm^3. Find the dimensions of the box that minimize the amount of material used.