Prince Sultan University
Orientation Mathematics Program

MATH 111

Final Exam

Semester I, Term 151

Saturday, December 26th, 2015

Time Allowed: **120 minutes**

Student Name: ____

Student ID #: ____ Section #: ______

Teacher's Name: **Dr. Aiman Mukheimer,** **Dr. Jamiiru Luttamaguzi**

Dr. Muhammad Dure, **Dr. Kamaleldin Abodayeh**

Serial Class Number: ______

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 12 problems, some with several parts.

Problems	Max points	Student's Points
1,2,3	18	
4,5,6,7	18	
8	12	
9,10,11	16	
12	16	
Total	**80**	**________% = ________/40**

1. (8 points) Find the derivative y' of each problem below simplifying where possible.

   (a) $y \sec x = x \tan y$

   (b) $y = \sin^{-1}(e^{9x}) - \coth(\ln x)$

2. (5 points) Given that the limit below represents a derivative of a function f at a point $x = a$.

   $\lim_{x \to 1} \frac{(3x^2 - 1)^3 - 8}{x - 1}$

   (a) What is the function f **and** the point $x = a$?

   (b) What is $f'(a)$ and $f''(a)$?

3. (5 point) Find the absolute maximum and absolute minimum values of the function $f(x) = (x^2 - 1)^3$ on the interval $[0, 2]$.

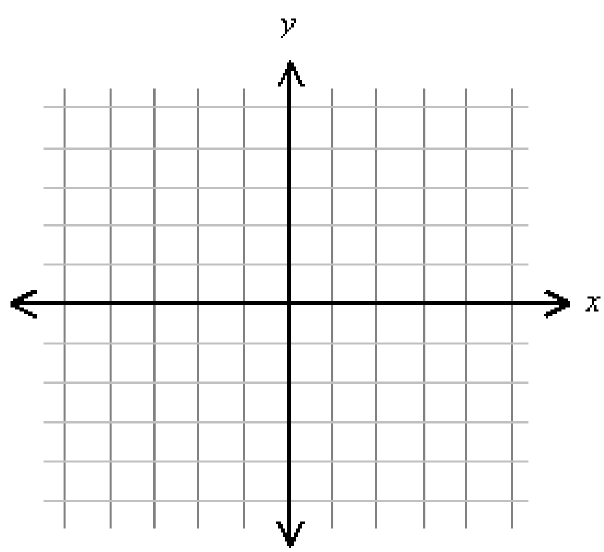
4. (5 points) If a snowball melts so its surface area decreases at a rate of 2 cm^2/min, find the rate at which the radius decreases when the diameter is 12 cm.

5. (5 points) Verify that the function $f(x) = \sqrt{x-3} - \frac{1}{3}x + 1$ satisfies the three hypotheses of Roll's Theorem on the interval $[3,12]$, then find the number c that satisfies the conclusion of Roll's Theorem.

6. (3 points) show that $\sqrt{\frac{1+\tanh x}{1-\tanh x}} = e^x$

7. (5 point) Sketch the graph of a function that satisfy the following conditions:

$$f'(0) = f'(2) = f'(4) = 0,$$
$$f'(x) > 0 \quad if \quad x < 0 \quad or \quad 2 < x < 4,$$
$$f'(x) < 0 \quad if \quad 0 < x < 2 \quad or \quad x > 4,$$
$$f''(x) > 0 \quad if \quad 1 < x < 3, \; f''(x) < 0 \quad if \quad x < 1 \quad or \quad x > 3$$



8. (12 points) Let $f(x) = \ln(9 - x^2)$

(a) (1 points) Find the domain of $f(x)$

(b) (1 points) Write down the y-intercept point.

(c) (1 points) Write down the x-intercept point(s).

(d) (1 points) Determine the vertical and horizontal asymptotes, if any.

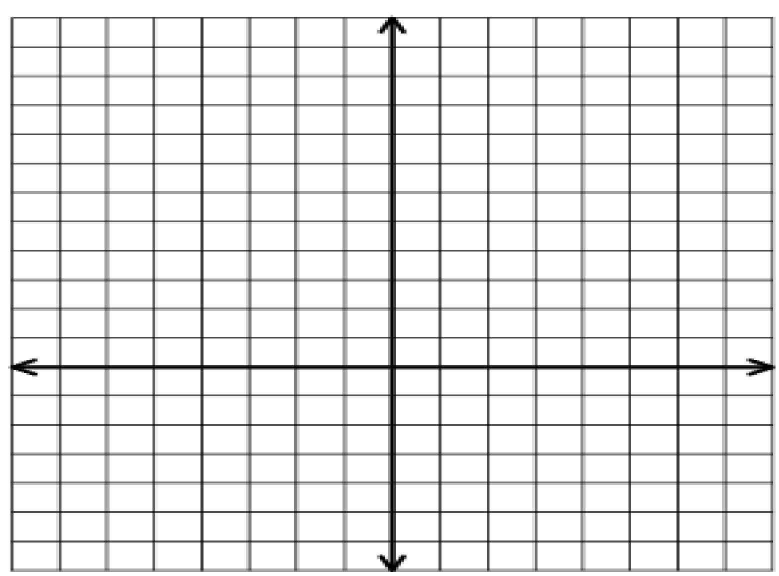
(e) (2 points) Find the critical numbers and the intervals on which f is increasing and/or decreasing.

(f) (1 points) Find the local maximum and/or local minimum points, if any.

(g) (1 points) Find the intervals on which f is concave up and/or concave down.

(h) (1 points) Find the inflection point(s) of the curve, if any.

(i) (3 points) Sketch the graph of f showing on the graph all significant features.



9. (6 points) A metal box tank with an **open top** needs to hold 500 liters (500,000 cm^3) of water. The tank has a **square bottom**. What are the **dimensions** that **minimize** the surface area of the metal used?

10. (4 points) Suppose that $f(-1) = 2$ and $f'(x) \le 4$ for all x such that $-1 \le x \le 3$. What is the largest possible value of $f(3)$? (Hint: you may use the Mean Value Theorem)

11. (6 points) Find the area of the largest rectangle that can be inscribed in a quarter of a circle of radius 16.

12. (16 points) Use L'Hospital's rule to evaluate the limits below (show your work in details).

(a) $\lim_{x\to 0}\dfrac{e^{2x^2}-1}{\sin(2x^2)}$

(b) $\lim_{x\to 0}\dfrac{\sinh x - x}{2x^3}$

(c) $\lim_{x\to 0}(1-4x)^{\frac{1}{x}}$

(d) $\lim_{x\to 0^+}\left(\dfrac{1}{x}-\dfrac{1}{e^x-1}\right)$