Prince Sultan University
Orientation Mathematics Program

MATH 111

Final Exam

Semester I, Term 141

Saturday, January 3rd, 2015

Time Allowed: **<u>120 minutes</u>**

Student Name: ____

Student ID #: ____ Section #: ______

Teacher's Name: **Dr. Aiman Mukheimer** and **Dr. Jamiiru Luttamaguzi**

Serial Number: [ ]

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination has 12 problems, some with several parts.

Problems	Max points	Student's Points
1	15	
2,3,4	18	
5	18	
6,7,8	19	
9	15	
10,11,12	15	
Total	**100**	**________% = ________/40**

1. (15 points) Find the derivative y' of each problem below simplifying where possible.

   (a) $\sin(2x)+\cos(2y)=xy$

   (b) $y=\ln(\sin^{-1}(2x))+\tan^{-1}(\ln(2x))$

   (c) $y=\sqrt{\dfrac{\sqrt[3]{2x}\tan^2 x}{(2+x)^3}}$ (using logarithmic differentiation)

2. (9 points) A cylindrical metal can (**<u>with an open top</u>**) needs to hold 1500 cm^3 of oil. What are the dimensions (radius, height) of the can that will minimize the surface area of the metal used? (Note: metal is used to make the side and the bottom of the can).

3. (5 points) Find the equations of the tangent line to the curve $y = \sec 3x$ at $x = \frac{\pi}{3}$.

4. (4 points) First verify that $\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}$ **<u>and</u>** then use it to show that: $\tanh(\ln x) = \frac{x^2 - 1}{x^2 + 1}$

5. (18 points) Consider the polynomial given by the equation: $y = 9(x-1)^2 - 3(x-1)^3$

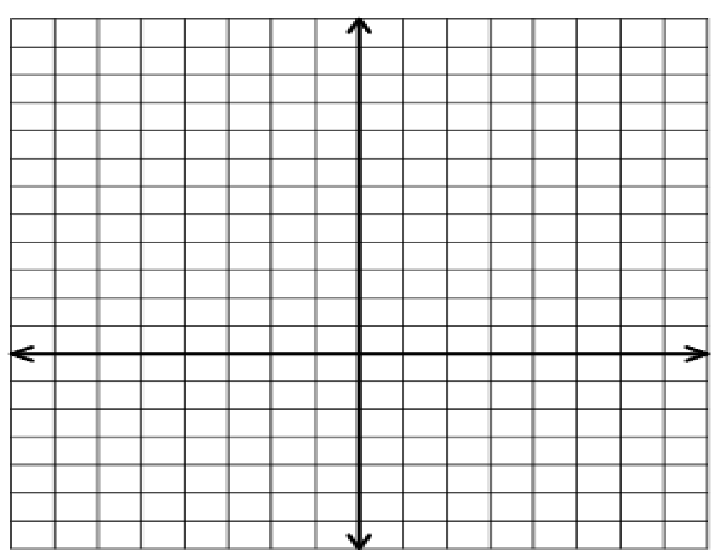
(a) (2 points) Write down the y-intercept point.

(b) (3 points) Write down the x-intercept point(s).

(c) (6 points) Find the critical point(s) of the curve. Find the regions of increase and decrease and use them to classify any local maximum and local minimum points.

(d) (3 points) Find the inflection point(s) of the curve. What are concavity upwards and concavity downward regions?

(e) (4 points) Graph the polynomial using the information above, showing on the graph all significant points.



6. (8 points) A water tank has the shape of an **inverted circular cone** with base radius 3 meters and height 4 meters. If the level of the water is observed to rise at the rate of 0.4 meters/min when the height is 1 meter, at what rate is the water being pumped at that instant (to 1 decimal point)?

7. (5 points) Let $f(x)=x^{\frac{2}{3}}-2$, Show that $f(-1)=f(1)$, **and** also show that there is no point c for which $f'(c)=0$. Why doesn't this contradict Rolle's Theorem?

8. (6 points) What is the domain of the function $f(x)=\dfrac{x}{x+2}$ and what is the domain of its derivative.
Verify that $f(x)$ satisfies the hypotheses of the Mean Value Theorem on the interval $D=[1,4]$.
Find all numbers c that satisfy the conclusion of the Mean Value Theorem.

9. (15 points) Use L'Hopital's rule to evaluate the limits below (show working).

(a) $\lim_{x\to 1} \frac{\sin \pi x}{\ln x}$

(b) $\lim_{x\to\infty} \left(1+\frac{2}{x}\right)^{3x}$

(c) $\lim_{x\to 1^+} \left(\ln(x^3-1)-\ln(x^2-1)\right)$

10. (6 points) What is the domain of the function $f(x)=\sqrt{x-x^2}$? Use The Extreme Value Theorem to find the absolute maximum value of $f(x)$ on the domain.

11. (4 points) The function f has $f(1)=-1$ and $f'(1)=-2$. Let $h(x)=\sqrt{3-4f(x)}$. Find the slope of the tangent to $h(x)$ at $x=1$

12. (5 points) Let $f(x)=\pi\cos(\pi x-e)$. Find its 2015$^{\text{th}}$ derivative $f^{(2015)}(x)$.