PRINCE SULTAN UNIVERSITY
Department of Mathematical Sciences

MATH 002 Final Examination
Saturday, 11 June 2005
(042)

Time allowed: 150 minutes

Student Name: ______________________________

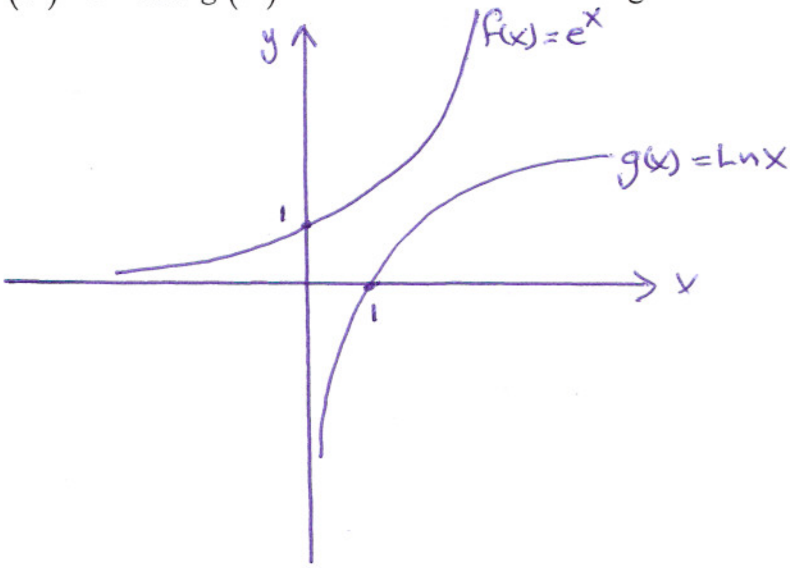
Student ID number: Answer Key

Section: ______________

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. If your mobile phone is seen or heard, your exam will be taken immediately.
6. You must show all your work beside the problem. Be organized.
7. You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.
8. This examination has 22 problems, one with two parts. Make sure your paper has all these problems.

Problems	Max points	Student's Points
1,2,3,4	15	
5,6,7,8	16	
9,10,11,12	14	
13,14	11	
15,16,17	13	
18,19	13	
20,21,22	18	
Total	100	

1. (4 points) Graph: $f(x)=e^x$ and $g(x)=\ln x$ in the same rectangular coordinate system.



2. (4 points) Write: $\frac{1}{2}(\log x+\log y)-2\log(x+1)$ as a single logarithm whose coefficient is one.

$$= \frac{1}{2}\log xy - 2\log(x+1)$$

$$= \log\sqrt{xy} - \log(x+1)^2$$

$$= \log\left[\frac{\sqrt{xy}}{(x+1)^2}\right]$$

3. (4 points) Solve: $4\ln 3x = 8$. Use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.

$4\ln 3x = 8$

$\ln 3x = 2$ take e for both sides

$3x = e^2$

$x = \frac{1}{3}e^2 = 2.46$

4. (3 points) A circle has a radius of 16 inches. Find the length of the arc intercepted by a central angle of 60°.

$$60^\circ = \frac{60^\circ * \pi}{180^\circ} = \frac{\pi}{3} \text{ rad}$$

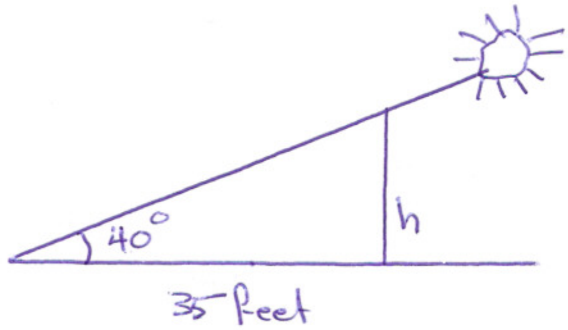
$$s = r\theta = 16 * \frac{\pi}{3} = \frac{16\pi}{3} = 16.75 \text{ inches}$$

5. (4 points) At a certain time of day, the angle of elevation of the sun is 40°. To the nearest foot, find the height of a tree whose shadow is 35 feet long.

$\tan 40^\circ = \frac{h}{35}$

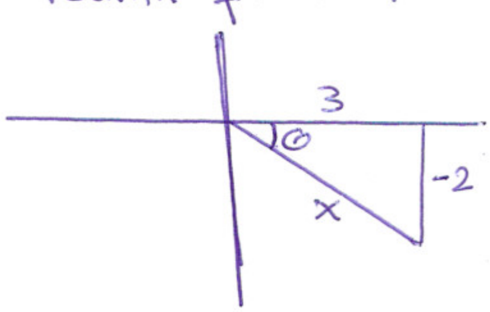
$h = 35 * \tan 40^\circ$

$= 29$ feet



6. (4 points) Given $\tan\theta = -\frac{2}{3}$ and $\cos\theta > 0$, find $\csc\theta$.

Fourth quadrant



$x^2 = 3^2 + (-2)^2 = 9 + 4 = 13$

$x = \sqrt{13}$

$\csc\theta = \frac{\sqrt{13}}{-2}$

7. (3 points) Find the **exact** value of $\cot 960^\circ$.

$960^\circ - 2*360^\circ = 240^\circ$ (third quadrant)

$\theta_{ref} = 240^\circ - 180^\circ = 60^\circ$

$\cot 60^\circ = \frac{\sqrt{3}}{3}$

$\cot 960^\circ = + \frac{\sqrt{3}}{3}$

8. (5 points) Graph one period of the function $y = -\sin\frac{2}{3}x$. Show the coordinates of the key points on the graph.

$A = -1 \quad B = \frac{2}{3} \quad C = 0$

Amplitude $= |A| = 1$

period $= \frac{2\pi}{B} = \frac{2\pi}{\frac{2}{3}} = 3\pi$

phase shift $= 0$

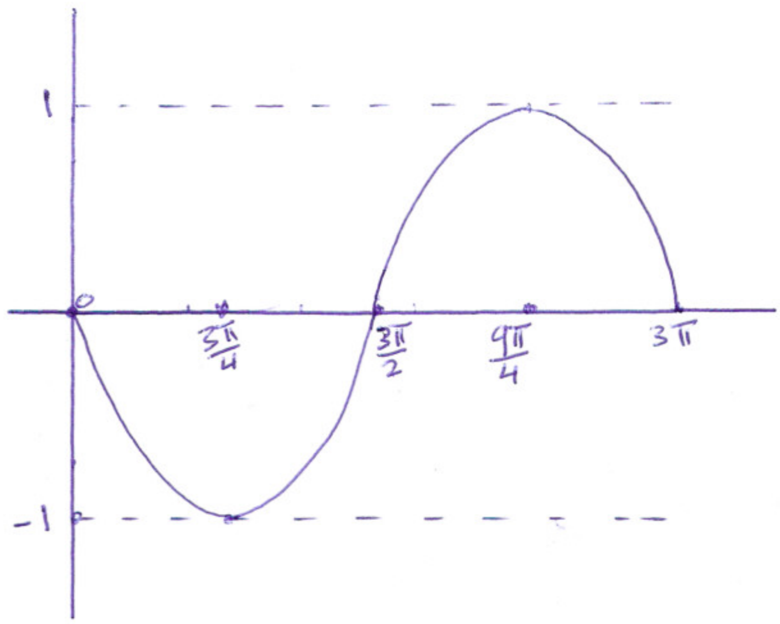
$x_1 = 0$

$x_2 = 0 + \frac{3\pi}{4} = \frac{3\pi}{4}$

$x_3 = \frac{3\pi}{4} + \frac{3\pi}{4} = \frac{3\pi}{2}$

$x_4 = \frac{3\pi}{2} + \frac{3\pi}{4} = \frac{9\pi}{4}$

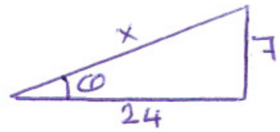
$x_5 = \frac{9\pi}{4} + \frac{3\pi}{4} = 3\pi$



9. (4 points) Find the **exact** value of $\sin\left(\tan^{-1}\dfrac{7}{24}\right)$.

let $\theta = \tan^{-1}\dfrac{7}{24}$

$\tan\theta = \dfrac{7}{24}$



$x = \sqrt{24^2 + 7^2} = 25$

$\sin\left(\tan^{-1}\dfrac{7}{24}\right) = \sin\theta = \dfrac{7}{25}$

10. (3 points) Verify the identity: $\cos(\alpha+\beta)+\cos(\alpha-\beta) = 2\cos\alpha\cos\beta$.

$L.H.S = \cos(\alpha+\beta) + \cos(\alpha-\beta)$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta + \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \cos\alpha\cos\beta + \cos\alpha\cos\beta$

$= 2\cos\alpha\cos\beta$

11. (4 points) Find the **exact** value of $\sin 75^\circ$.

$\sin 75^\circ = \sin(30^\circ + 45^\circ)$

$= \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$

$= \dfrac{1}{2}\cdot\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{2}}{4} + \dfrac{\sqrt{6}}{4} = \dfrac{\sqrt{2}+\sqrt{6}}{4}$

12. (3 points) Find all solutions of the equation $3\sin\theta + 5 = -2\sin\theta$.

$3\sin\theta + 5 = -2\sin\theta$

$5\sin\theta = -5$

$\sin\theta = -1$

$\theta = \dfrac{3\pi}{2} + 2n\pi$

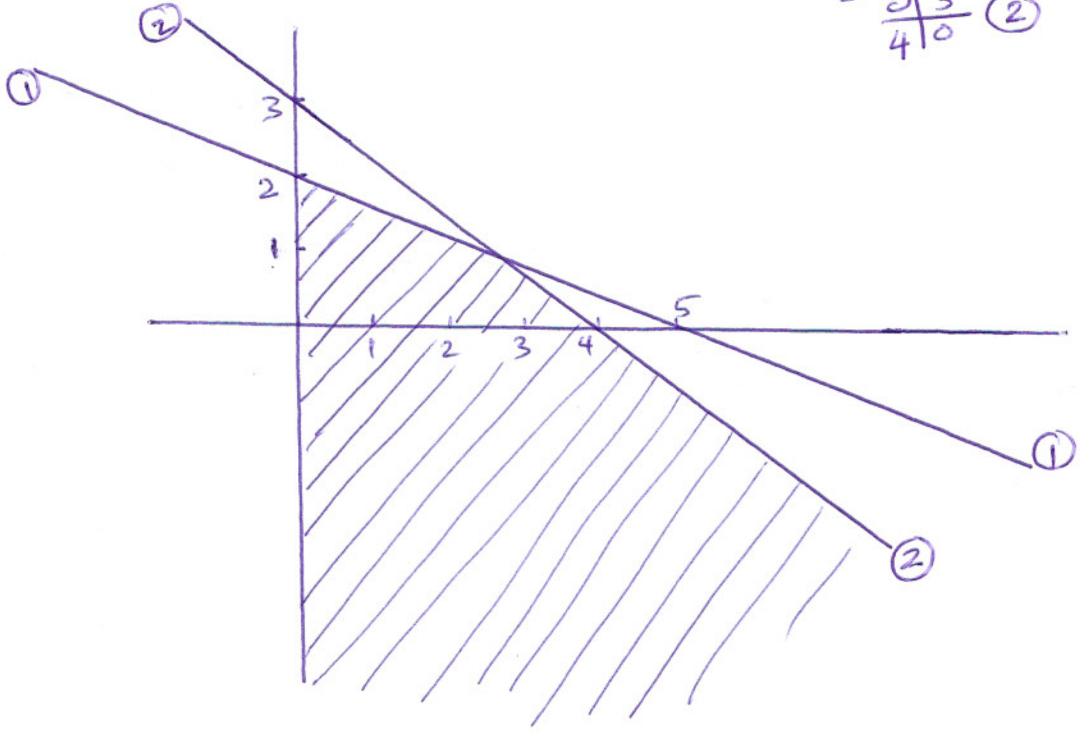
13. (6 points) Graph the solution set of the system: $\begin{aligned} x &\geq 0 \\ 2x + 5y &\leq 10. \\ 3x + 4y &\leq 12 \end{aligned}$

① $2x + 5y = 10$:

x	y
0	2
5	0

② $3x + 4y = 12$:

x	y
0	3
4	0



14. (5 points) Solve the system: $\begin{aligned} x + z &= 2 \quad \cdots\cdots eq(1) \\ x + 2y - z &= 3 . \quad \cdots\cdots eq(2) \\ 2x - y + 3z &= 5 \quad \cdots\cdots eq(3) \end{aligned}$

$$\begin{aligned} eq2 &: \quad x + 2y - z = 3 \\ 2 * eq3 &: \quad 4x - 2y + 6z = 10 \\ \hline & \quad 5x + 5z = 13 \quad \cdots\cdots eq(4) \end{aligned}$$

$$\begin{aligned} eq(4) &: \quad 5x + 5z = 13 \\ -5 * eq(1) &: \quad -5x - 5z = -10 \\ \hline & \quad 0 = 3 \end{aligned}$$

the system in inconsistent (has no solution)

15. (5 points) Write the augmented matrix of the system: $\begin{aligned} x + y + 2z &= 19 \\ -2y - 4z &= -26 \\ 2y &= 6 \end{aligned}$. Then perform the following two row operations.

(a) $-\frac{1}{2}R_2$ (b) $-2R_2 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & -2 & -4 & -26 \\ 0 & 2 & 0 & 6 \end{array}\right]$$

a) $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & 1 & 2 & 13 \\ 0 & 2 & 0 & 6 \end{array}\right]$

b) $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & -2 & -4 & -26 \\ 0 & 6 & 8 & 58 \end{array}\right]$

16. (4 points) Find the solution set of the system: $\begin{aligned} 3x - y + 4z &= 8 \\ y + 2z &= 1 \end{aligned}$.

$y = 1 - 2z$

$3x - (1 - 2z) + 4z = 8$

$3x - 1 + 2z + 4z = 8$

$3x - 1 + 6z = 8$

$3x = 9 - 6z$

$x = 3 - 2z$

$(3 - 2z,\ 1 - 2z,\ z)$

17. (4 points) Perform the matrix operation $AB - 2A$ given that $A = \begin{bmatrix} 4 & 0 \\ -3 & 5 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 1 \\ -2 & -2 \end{bmatrix}$.

$$AB = \begin{bmatrix} 4 & 0 \\ -3 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ -2 & -2 \end{bmatrix} = \begin{bmatrix} 20 & 4 \\ -25 & -13 \\ -2 & -2 \end{bmatrix}$$

$$AB - 2A = \begin{bmatrix} 20 & 4 \\ -25 & -13 \\ -2 & -2 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -6 & 10 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 12 & 4 \\ -19 & -23 \\ -2 & -4 \end{bmatrix}$$

18. (8 points) Let $A = \begin{bmatrix} 2 & 6 & 1 \\ 0 & -1 & -1 \\ -1 & -2 & 1 \end{bmatrix}$. Find A^{-1}. Check that $AA^{-1} = I$ and $A^{-1}A = I$.

$$\left[\begin{array}{ccc|ccc} 2 & 6 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ -1 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3+R_1} \left[\begin{array}{ccc|ccc} 1 & 8 & 2 & 1 & 0 & 1 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ -1 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1+R_3} \left[\begin{array}{ccc|ccc} 1 & 8 & 2 & 1 & 0 & 1 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 0 & 10 & 3 & 1 & 0 & 2 \end{array}\right]$$

$$\xrightarrow{-R_2} \left[\begin{array}{ccc|ccc} 1 & 8 & 2 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 10 & 3 & 1 & 0 & 2 \end{array}\right] \xrightarrow[-10R_2+R_3]{-8R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & -6 & 1 & 8 & 1 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & -7 & 1 & 10 & 2 \end{array}\right]$$

$$\xrightarrow{\frac{R_3}{-7}} \left[\begin{array}{ccc|ccc} 1 & 0 & -6 & 1 & 8 & 1 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{7} & \frac{-10}{7} & \frac{-2}{7} \end{array}\right] \xrightarrow[-R_3+R_2]{6R_3+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{7} & \frac{-4}{7} & \frac{-5}{7} \\ 0 & 1 & 0 & \frac{1}{7} & \frac{3}{7} & \frac{2}{7} \\ 0 & 0 & 1 & -\frac{1}{7} & \frac{-10}{7} & \frac{-2}{7} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} \frac{1}{7} & \frac{-4}{7} & \frac{-5}{7} \\ \frac{1}{7} & \frac{3}{7} & \frac{2}{7} \\ -\frac{1}{7} & \frac{-10}{7} & \frac{-2}{7} \end{bmatrix}, \quad AA^{-1} = \begin{bmatrix} 2 & 6 & 1 \\ 0 & -1 & -1 \\ -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{7} & \frac{-4}{7} & \frac{-5}{7} \\ \frac{1}{7} & \frac{3}{7} & \frac{2}{7} \\ -\frac{1}{7} & \frac{-10}{7} & \frac{-2}{7} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

19. (5 points) Evaluate each determinant.

(a) $\begin{vmatrix} 1 & -3 \\ -8 & 2 \end{vmatrix} = (1*2) - (-8*{}^{-}3) = 2 - 24 = -22$

(b) $\begin{vmatrix} 2 & -4 & 2 \\ -1 & 0 & 5 \\ 3 & 0 & 4 \end{vmatrix} = -(-4)\begin{vmatrix} -1 & 5 \\ 3 & 4 \end{vmatrix} = 4\left[(-1*4) - (5*3)\right]$

$= 4\left[-4 - 15\right]$

$= 4 * {}^{-}19$

$= -76$

20. (5 points) Find the vertex, focus, and directrix of the parabola given by $y^2 + 2y + 4x - 7 = 0$.

$y^2 + 2y = -4x + 7$

$y^2 + 2y + 1 = -4x + 8$

$(y+1)^2 = -4(x-2)$

$p = 1$

vertex $(2, -1)$

focus $(1, -1)$

directrix $x = 3$

21. (8 points) Graph: $\dfrac{(x-3)^2}{4} - \dfrac{(y-1)^2}{1} = 1$. Where are the foci located? What are the equations of the asymptotes?

$a^2 = 4 \Rightarrow a = 2$

$b^2 = 1 \Rightarrow b = 1$

$c^2 = 4 + 1 = 5$

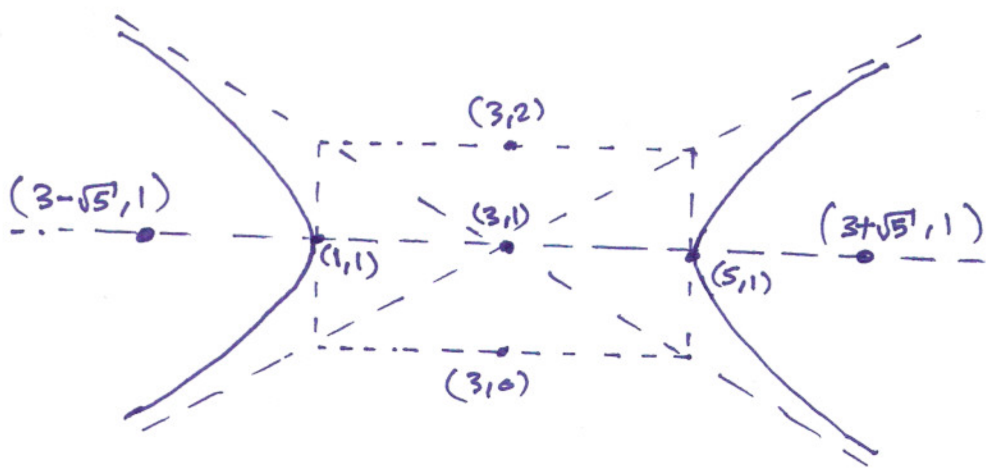
$c = \sqrt{5}$

foci: $(3 \pm \sqrt{5}, 1)$

asymptotes

$(y-1) = \pm \frac{b}{a}(x-3)$

$(y-1) = \pm \frac{1}{2}(x-3)$



22. (5 points) Find the standard form of the equation of the ellipse with endpoints of major axis at (2,2) and (8,2) and endpoints of minor axis at (5,3) and (5,1).

$2a = 8 - 2 = 6 \Rightarrow a = 3$ major axis is horizontal

$2b = 3 - 1 = 2 \Rightarrow b = 1$

center $(5, 2)$

$$\frac{(x-5)^2}{9} + \frac{(y-2)^2}{1} = 1$$