PRINCE SULTAN UNIVERSITY
Department of Mathematical Sciences

MATH 002 Final Examination
Tuesday, 4 January 2005
(041)

Time allowed: 150 minutes

Student Name: ______________________

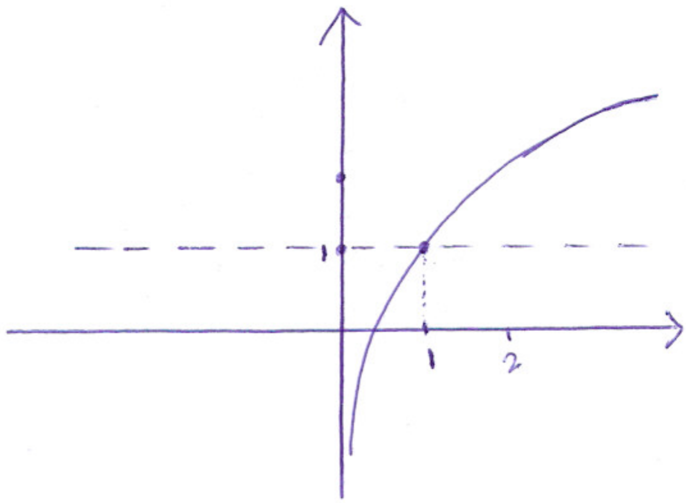
Student ID number: Answer Key

Section: ____________

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. If your mobile phone is seen or heard, your exam will be taken immediately.
6. You must show all your work beside the problem. Be organized.
7. You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.
8. This examination has 19 problems, some with several parts. Make sure your paper has all these problems.

Problems	Max points	Student's Points
1,2,3,4	20	
5,6,7,8	19	
9,10,11	15	
12,13,14	18	
15,16,17	16	
18,19	12	
Total	100	

1. (6 points) Sketch the graph of $f(x) = 1 + \log_3 x$. What is the graph's x-intercept? What is the vertical asymptote?



$1 + \log_3 x = 0$

$\log_3 x = -1$

$x = 3^{-1}$

$x = \frac{1}{3}$

x-intercept $= \frac{1}{3}$

vertical asymptote: $x = 0$

2. (6 points) Use properties of logarithms to expand $\log\left[\frac{10x^2\sqrt[3]{1-x}}{7(x+1)^2}\right]$ as much as possible.

$= \log 10 + \log x^2 + \log(1-x)^{\frac{1}{3}} - \log 7 - \log(x+1)^2$

$= 1 + 2\log x + \frac{1}{3}\log(1-x) - \log 7 - 2\log(x+1)$

3. (4 points) Solve $3^{x/7} = 0.2$.

$\log_3 3^{\frac{x}{7}} = \log_3 0.2 \Longrightarrow \frac{x}{7} = \log_3 0.2$

$x = 7\log_3 0.2 = -10.25$

4. (4 points) A circle has a radius of 10 inches. Find the length of the arc intercepted by a central angle of 120°.

$120° = \frac{120° * \pi}{180°} = \frac{2\pi}{3}$

$S = r\theta = 10 * \frac{2\pi}{3} = \frac{20\pi}{3} = 20.94$ inches

5. (5 points) Given $\sin\theta = \frac{1}{2}$ and $\cos\theta = \frac{-\sqrt{3}}{2}$, find the value of each of the four remaining trigonometric functions.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

$$\cot\theta = -\sqrt{3}$$

$$\sec\theta = -\frac{2}{\sqrt{3}}$$

$$\csc\theta = 2$$

6. (6 points) Find the **exact** value of each of the following

.

(i) $\cos 585^\circ = \cos(585^\circ - 360^\circ) = \cos(225^\circ)$ third quadrant

$$\theta_{ref} = 225^\circ - 180^\circ = 45^\circ$$

$$\cos 45^\circ = \frac{\sqrt{2}}{2} \Longrightarrow \cos 585 = -\frac{\sqrt{2}}{2}$$

(ii) $\cot\frac{-101\pi}{3} = \cot\left(\frac{-101\pi}{3} + 34\pi\right) = \cot\left(\frac{\pi}{3}\right)$

$$= \frac{1}{\tan\left(\frac{\pi}{3}\right)} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

7. (4 points) Find the exact value, in radian, of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

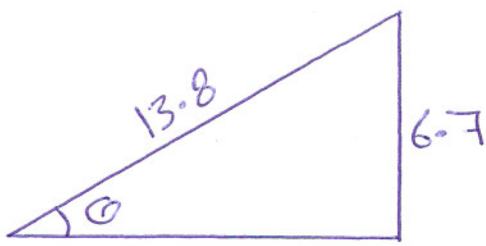
$$\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

8. (4 points) A guy wire is 13.8 yards long and is attached from the ground to a pole 6.7 yards above the ground. Find the angle, to the nearest tenth of a degree, that the wire makes with the ground.

$$\sin\theta = \frac{6.7}{13.8}$$

$$\theta = \sin^{-1}\left(\frac{6.7}{13.8}\right)$$

$$= 29.0^\circ$$



9. (6 points) Determine the amplitude, period, and phase shift of $y=-3\sin\left(2x-\frac{\pi}{2}\right)$. Then graph one period of the function locating key points on the graph. $A=-3$, $B=2$, $C=\frac{\pi}{2}$

Amplitude $=|A|=3$

period $=\frac{2\pi}{B}=\frac{2\pi}{2}=\pi$

phase shift $=\frac{C}{B}=\frac{\frac{\pi}{2}}{2}=\frac{\pi}{4}$

Key points

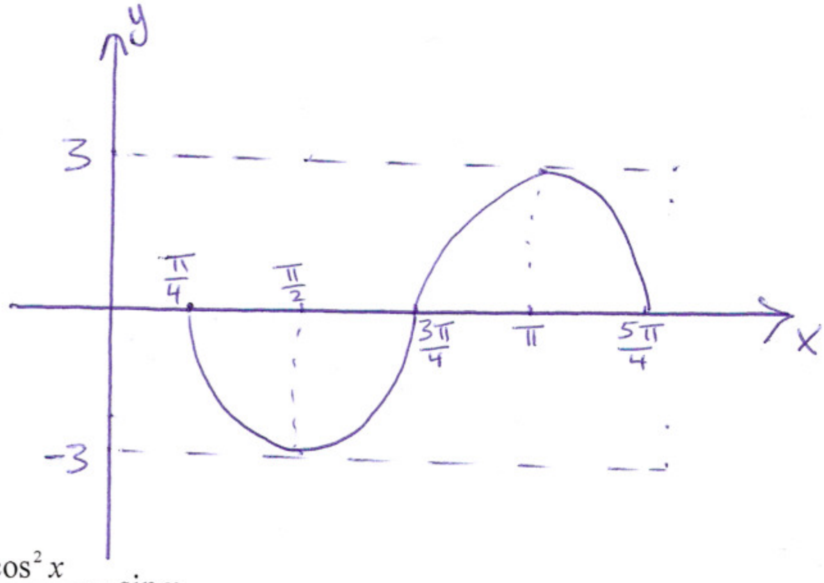
$x_1=\frac{\pi}{4}$

$x_2=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}$

$x_3=\frac{\pi}{4}+\frac{\pi}{2}=\frac{3\pi}{4}$

$x_4=\frac{\pi}{4}+\frac{3\pi}{4}=\pi$

$x_5=\frac{\pi}{4}+\pi=\frac{5\pi}{4}$



10. (4 points) Verify the identity: $1-\frac{\cos^2 x}{1+\sin x}=\sin x$.

$$L.H.S=1-\frac{\cos^2 x}{1+\sin x}=1-\frac{\cos^2 x}{1+\sin x}\times\frac{1-\sin x}{1-\sin x}$$

$$=1-\frac{\cos^2 x(1-\sin x)}{1-\sin^2 x}$$

$$=1-\frac{\cos^2 x\,(1-\sin x)}{\cos^2 x}$$

$$=1-(1-\sin x)=1-1+\sin x=\sin x=R.H.S$$

11. (5 points) Find the **exact** value of $\cos\frac{19\pi}{12}$.

$$\cos\frac{19\pi}{12}=\cos\left(2\pi-\frac{5\pi}{12}\right)=\cos\left(-\frac{5\pi}{12}\right)=\cos\frac{5\pi}{12}$$

$$\cos\frac{5\pi}{12}=\cos\left(\frac{2\pi}{12}+\frac{3\pi}{12}\right)$$

$$=\cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right)=\cos\frac{\pi}{6}\cos\frac{\pi}{4}-\sin\frac{\pi}{6}\sin\frac{\pi}{4}$$

$$=\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$

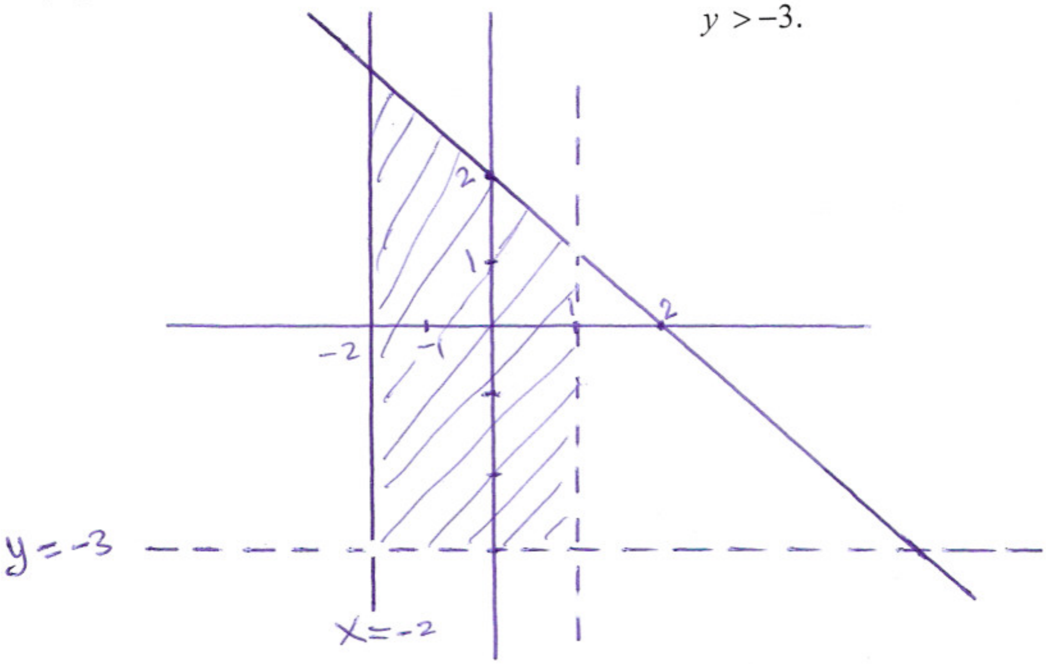
$$=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$$

$$=\frac{\sqrt{6}-\sqrt{2}}{4}$$

12. (5 points) Solve the equation $4\sin^2 x + 4\cos x - 5 = 0$ on the interval $[0, 2\pi)$.

$$4(1-\cos^2\theta) + 4\cos\theta - 5 = 0$$
$$4 - 4\cos^2\theta + 4\cos\theta - 5 = 0$$
$$-4\cos^2\theta + 4\cos\theta - 1 = 0$$
$$4\cos^2\theta - 4\cos\theta + 1 = 0$$
$$(2\cos\theta - 1)^2 = 0$$
$$2\cos\theta - 1 = 0$$
$$\cos\theta = \frac{1}{2} \Longrightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

13. (7 points) Graph the solution set of the system
$$x + y \le 2$$
$$-2 \le x < 1$$
$$y > -3.$$



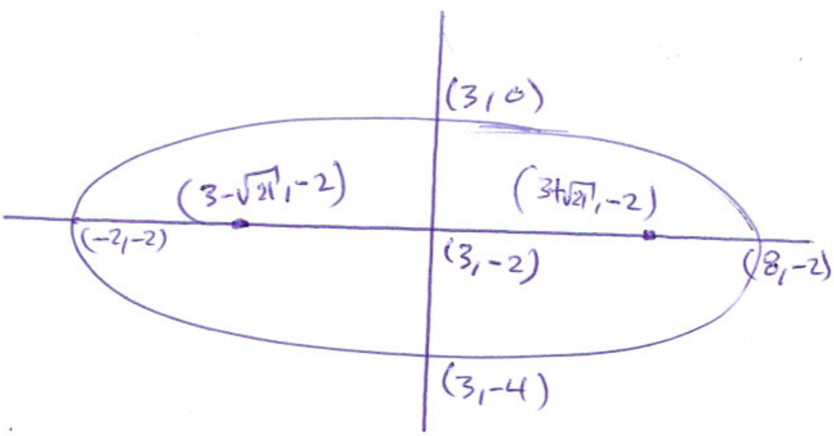
14. (6 points) Graph: $4x^2 + 25y^2 - 24x + 100y + 36 = 0$.

$$4(x^2-6x) + 25(y^2+4y) + 36 = 0$$
$$4(x^2-6x+9-9) + 25(y^2+4y+4-4) + 36 = 0$$
$$4(x-3)^2 + 25(y+2)^2 = 100$$
$$\frac{(x-3)^2}{25} + \frac{(y+2)^2}{4} = 1$$

ellipse $a^2 = 25 \Rightarrow a = 5$

$b^2 = 4 \Rightarrow b = 2$

$c^2 = 21 \Rightarrow c = \sqrt{21}$



15. (6 points) Find the standard form of the equation of the hyperbola satisfying the following conditions. : Center: $(4,-2)$; Focus: $(7,-2)$; vertex: $(6,-2)$.

transverse axis: horizontal

$a = 6-4 = 2$

$c = 7-4 = 3$

$b^2 = c^2 - a^2 = 9-4 = 5$

$$\frac{(x-4)^2}{4} - \frac{(y+2)^2}{5} = 1$$

16. (4 points) Find BA, given $A = [1,2,3]$ and $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$.

$$BA = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 12 \\ 5 & 10 & 15 \\ 6 & 12 & 18 \end{bmatrix}$$

17. (6 points) Solve the system $\begin{aligned} x + 3y &= 0 \\ x + y + z &= 1 \\ y - z &= 11 \end{aligned}$ using matrices. Use Gaussian elimination with back-substitution or Gauss-Jordan elimination.

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 11 \end{array}\right] \xrightarrow{-R_1+R_2} \left[\begin{array}{ccc|c} 1 & 3 & 0 & 0 \\ 0 & -2 & 1 & 1 \\ 0 & 1 & -1 & 11 \end{array}\right] \xrightarrow{3R_3+R_2} \left[\begin{array}{ccc|c} 1 & 3 & 0 & 0 \\ 0 & 1 & -2 & 34 \\ 0 & 1 & -1 & 11 \end{array}\right]$$

$$\xrightarrow[-R_2+R_3]{-3R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 6 & -102 \\ 0 & 1 & -2 & 34 \\ 0 & 0 & 1 & -23 \end{array}\right] \xrightarrow[2R_3+R_2]{-6R_3+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 36 \\ 0 & 1 & 0 & -12 \\ 0 & 0 & 1 & -23 \end{array}\right]$$

$x = 36$, $y = -12$, $z = -23$

check

$x + 3y = 36 + 3(-12) = 0$ true

$x + y + z = 36 - 12 - 23 = 1$ true

$y - z = -12 + 23 = 11$ true

18. (5 points) Find the multiplicative inverse of $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$.

See last page

19. (7 points) Use Cramer's rule to solve the system $\begin{aligned} x + 3y &= 0 \\ x + y + z &= 1. \\ y - z &= 11 \end{aligned}$

$$D = \begin{vmatrix} 1 & 3 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = (1)(-1-1) + (-3)(0-1) = -2+3 = 1$$

$$D_x = \begin{vmatrix} 0 & 3 & 0 \\ 1 & 1 & 1 \\ 11 & 1 & -1 \end{vmatrix} = (-3)(-1-11) = 36$$

$$D_y = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 11 & -1 \end{vmatrix} = (+1)(-1-11) = -12$$

$$D_z = \begin{vmatrix} 1 & 3 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 11 \end{vmatrix} = (+1)(11-1) + (-3)(11-0) = 10 - 33 = -23$$

$$x = \frac{D_x}{D} = \frac{36}{1} = 36$$

$$y = \frac{D_y}{D} = \frac{-12}{1} = -12$$

$$z = \frac{D_z}{D} = \frac{-23}{1} = -23$$

solution of question 18

$$\left[\begin{array}{cccc|cccc} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_4} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{-R_1+R_3} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_4 \leftrightarrow R_2} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \leftrightarrow R_4} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & -1 \end{array}\right]$$

$$\xrightarrow{-R_4} \left[\begin{array}{cccccccc} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right]$$

$$\xrightarrow{-R_4+R_1} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$