Prince Sultan University — Dept. of Mathematical Sciences

Date. April 2013 — Math 215- MidTerm No.2 -

1. Use Lagrange multipliers to minimize $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraints $g(x, y, z) = x + y + z - 12 = 0$ and $h(x, y, z) = x^2 + y^2 - z = 0$

2. Find local extrema for $f(x, y) = 2x^2 - y^3 - 2xy$.

3. The points $A(4, 5, 2)$, $B(1, 7, 3)$ and $C(2, 4, 5)$ are they vertices of an equilateral triangle ?

4. Find the directional derivative of $f(x, y, z) = y^2 + 2ye^{4x}$ at $P(0, -2)$ in the direction from $(0, -2)$ to $(-4, 4)$.

5. Prove that $\|u + v\|^2 + \|u - v\|^2 = 2\|u\|^2 + 2\|v\|^2, \quad u, v \in \mathbb{R}^3$.

6. Determine if the following vectors are coplanar ?

$$\overrightarrow{a} = \overrightarrow{i} - 2\overrightarrow{j} + \overrightarrow{k}, \quad \overrightarrow{b} = 3\overrightarrow{i} - 2\overrightarrow{k}, \quad \overrightarrow{c} = 5\overrightarrow{i} - 4\overrightarrow{j}.$$

7. Find a vector $\overrightarrow{a}$ perpendicular to the plane determined by the following points $A(0, -2, 1)$, $B(1, -1, -2)$, and $C(-1, 1, 0)$.

8. If the four vectors $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ and $\overrightarrow{d}$ are coplanar, show that

$$(\overrightarrow{a} \times \overrightarrow{b}) \times (\overrightarrow{c} \times \overrightarrow{d}) = 0.$$

9. Find k_1, k_2 such that the point $P(k_1, 1, k_2)$ lies on the same line passing through $Q(0, 2, 3)$ and $R(2, 7, 5)$.

10. Find an equation of the plane that passes through $P(-1, 2, 1)$ and $Q(1, -1, 2)$ and is parallel to the line of intersection of the planes :

$$2x - 3y - z = 6, \quad \text{and} \quad 3x - y + 2z = 0.$$