Prince Sultan University — Dept. of Mathematical Sciences
Date. May 27, 2013 — Math 215- Final Exam-

1. Find $\displaystyle\int\int_R \frac{1}{\sqrt{x^2+y^2+9}}dA$ where R is the region bounded by the triangle with vertices $(0,0),\ (3,0),\ (3,3)$ by using polar coordinates.

2. Evaluate the integral
$$\int_0^2 \int_{y^2}^4 y^3 e^{x^3}\,dxdy.$$

3. Express the following integral
$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{\sqrt{1-x^2-y^2-z^2}\,dxdydz}{x^2+y^2+z^2}$$
using spherical coordinates.

4. Determine each limit
$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}, \qquad \lim_{(x,y)\to(1,2)} \frac{x^2-xy+y-1}{x^2+xy-3x-y+2}$$

5. Find $f_x,\ f_y$ and f_z of $f(x,y,z) = \left(\dfrac{xz}{1-z^2-y^2}\right)^{-3/4}$.

6. Let $z = f(x,y)$ be a function with continuous partial derivatives, where $x = s+2t$ and $y = 3s-t$. Find $\dfrac{\partial^2 z}{\partial s^2}$.

7. Find the local extrema and saddle points, if any, of the function $f(x,y) = 3xy - x^2y - xy^2$.

8. Let $\overrightarrow{u}$ be any unit vector. Show that $|D_{\overrightarrow{u}} f(x,y)| \leq e\sqrt{2}$, where $f(x,y) = e^{\cos(x)\sin(y)}$.

9. Prove that $|\overrightarrow{u} \bullet \overrightarrow{v}|^2 + \|\overrightarrow{u} \times \overrightarrow{v}\|^2 = \|\overline{u}\|^2 \times \|\overrightarrow{v}\|^2, \quad \overrightarrow{u}, \overrightarrow{v} \in \mathbb{R}^3$.

10. Show that the lines : $L_1 : x = -2+t,\ y = 3+2t,\ z = 4-t$ and $L_2 :\ x = 3-t,\ y = 4-2t,\ z = t$ are parallel and find the equation of the plane they determine.

11. Determine if the following vectors are coplanar ? [Explain your method and give the arguments of your solution]
$$\overrightarrow{a} = 2\overrightarrow{i} - 4\overrightarrow{j} + \overrightarrow{k}, \quad \overrightarrow{b} = -5\overrightarrow{i} + 2\overrightarrow{k}, \quad \overrightarrow{c} = -3\overrightarrow{i} + 2\overrightarrow{j}.$$

12. Let the matrix A :

$$A = \begin{pmatrix} 2 & 0 & 0 \\ -3 & 2 & -1 \\ 9 & 0 & 5 \end{pmatrix}$$

(a) Find a basis for the Column space of A.

(b) Determine the rank of A.

(d) Find if possible, a non-singular matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

13. If u and v are such that : $(\overrightarrow{u} + 2\overrightarrow{v}) \bullet (\overrightarrow{u} - \overrightarrow{v}) = 0$ and $(2\overrightarrow{u} + 3\overrightarrow{v}) \bullet (\overrightarrow{u} - \lambda\overrightarrow{v}) = 0$ where $\lambda \in \mathbb{R}^*$. Determine $\dfrac{\overrightarrow{u} \bullet \overrightarrow{v}}{\|\overrightarrow{v}\|^2}$ in terms of λ.