Prince Sultan University, Department of Mathematics, 6/11/10. Calculus II (Math 113), Mid-Term 1- Fall (101).

Duration: 90 min

Dr Abdul-Wahed HAMDI

- Answer all the questions.
- This exam consists of a total of 3 pages and 9 questions.
- Show your working in the space provided for each question.
- Show all the key steps of your work.
- Scientific, non-programmable calculators are allowed.

Question	Maximum Score	Your Score
1	9	
2	12	
3	4	
4	5	
5	5	
6	6	
7	7	
8	5	
9	7	
Total	60	

Name	
ID Number	

1. Find the following antiderivatives

(a)
$$\int x\sqrt{2+x} \ dx$$

(b)
$$\int \cos^2(3x) \ dx$$

(c)
$$\int \cot(1-x) \ dx$$

2. Evaluate the following definite integrals (a) $\int_{-1}^{2} \frac{x \ dx}{4 + 5x^2}$

(a)
$$\int_{-1}^{2} \frac{x \, dx}{4 + 5x^2}$$

(b)
$$\int_{-1}^{2} \sqrt{2+|x|} \ dx$$

(c)
$$\int_0^{a/b} \sqrt{a^2 - b^2 x^2} \ dx$$
.

3. Solve the initial-value problem

$$\frac{dy}{dx} = \sin(x) + 1, \quad y(\pi/3) = \frac{1}{2}.$$

4. Evaluate $\sum_{j=4}^{20} j(1-j)^2$

5. Find the limit by interpreting it as a limit of a Riemann sum in which the interval [0, 1] is divided into n subintervals of equal length

$$\lim_{n \to \infty} \frac{1^4 + 2^4 + 3^4 + \dots + n^4}{n^5}.$$

6. Use the limit of summation to find the area under the curve of f(x) = 2 + 4x over the interval [0, 3]. Use x_k^* as the midpoint of each subinterval.

7. Find the total area of the region between the x-axis and the graph of $y = x^2 - 3x$ over the interval [-1, 4].

8. A particle moves with a velocity $v(t) = 0.5 - \frac{1}{t^2} \ m/s$ along an s-axis. Find the displacement and the distance traveled by the particle during the time interval [1, 3].

- **9.** Find
 - (a) the value(s) of x^* that satisfies the Mean-Value Theorem for the function $\phi(x) = 2x x^2$ over the interval [0, 2],

- (b) the average value of ϕ on [0, 2],
- (c) F'(x) and its domain where $F(x) = \int_4^x \frac{1}{\sqrt{t}}$.
- **10. Bonus question** Show that $\int_0^{\pi/2} \sin^n(x) dx = \int_0^{\pi/2} \cos^n(x) dx$.