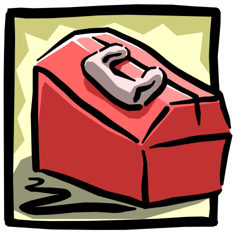
WORKING WITH LOGARITHMS



MAT 110 Toolbox Series

TOOLS NEEDED FOR THESE PROBLEMS

What is a logarithm?

b^y=x is an **exponential** equation. The base is **b**, and **y** is the base's exponent.

When *b* is any positive number other than 1, there is an alternate way to write this equation. **$y=\log_b x$** is the **logarithmic** form of the equation. In order for this logarithmic form to be valid, there are certain requirements.

1. **b** must be a positive number, but it cannot be equal to 1.
2. **x** must be a positive number. (It cannot be negative or zero.)

Properties of logarithms	For logs of any base	For base e logs
Multiplication property	$\log_b xy = \log_b x + \log_b y$	$\ln xy = \ln x + \ln y$
Division property	$\log_b \frac{x}{y} = \log_b x - \log_b y$	$\ln \frac{x}{y} = \ln x - \ln y$
Power property	$\log_b x^y = y \log_b x$	$\ln x^y = y \ln x$
Change of base formula	$\log_b x = \frac{\log_a x}{\log_a b}$	$\log_b x = \frac{\ln x}{\ln b}$
Inverse properties	$\log_b b^x = x$ $b^{\log_b x} = x$	$\ln e^x = x$ $e^{\ln x} = x$

Examples

I. A. Convert the logarithmic equation $\log_8 64 = 2$ into an exponential equation.

8 is the base for the log. When converting from log form to exponential form, the base for the log is also the base for the exponent.

2, the log, is also the exponent when the relationship is written in exponential form.

8 is the base for the log and the base for the exponent.

$\log_8 64 = 2$

$8^2 = 64$

2 is the log, and it is the exponent for the base of 8 in the exponential form.

B. Convert the exponential equation $3^4 = 81$ into a logarithmic equation.

To do this conversion, you are reversing the process in part A.

3 is the base for the exponent and the base for the log.

$3^4 = 81$

$\log_3 81 = 4$

4 is the exponent for the base of 3, and it is the log.

II. Use the change of base formula to convert these logs into base 10 logs and into natural logs. Then use a calculator to evaluate the logs.

	Rewrite as base 10 log	Rewrite as natural log (base e)	Calculator's answer
A. $\log_3 14$	$\log_3 14 = \frac{\log 14}{\log 3}$	$\log_3 14 = \frac{\ln 14}{\ln 3}$	**$\log_3 14 = 2.402173503$**
B. $\log_7 5$	$\log_7 5 = \frac{\log 5}{\log 7}$	$\log_7 5 = \frac{\ln 5}{\ln 7}$	**$\log_7 5 = .8270874753$**

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III. Solve this equation $5^{x-2} = 41$ by using logarithms.

$5^{x-2} = 41$

Begin by taking the log of both sides of the equation.

Use the **power rule** to change the exponent of (x-2) into a multiplier of log 5.

$\log 5^{x-2} = \log 41$

$(x-2)\log 5 = \log 41$

Distribute (multiply) the binomial (x - 2) by log 5.

$x\log 5 - 2\log 5 = \log 41$

$x\log 5 = \log 41 + 2\log 5$

$$\frac{x\log 5}{\log 5} = \frac{\log 41 + 2\log 5}{\log 5}$$

$$x = \frac{\log 41 + 2\log 5}{\log 5}$$

The **power property** allows the coefficient of the log to become the exponent of the argument.

$$x = \frac{\log 41 + \log 5^2}{\log 5}$$

$$x = \frac{\log 41 + \log 25}{\log 5}$$

At this point, the numerator is the sum of two logs.

$$x = \frac{\log[(41)(25)]}{\log 5}$$

Use the **multiplication property** to turn the two logs into one log .

$$x = \frac{\log 1025}{\log 5}$$

$x \approx 4.30737$

IV. A. Use the properties for logarithms to rewrite $\log\frac{x^3y^5\sqrt{z}}{wv^2}$ as the sum or difference of logarithms.

$$\log\frac{x^3y^5\sqrt{z}}{wv^2}$$

The **division property** allows you to break the original expression into the log of the numerator and the log of the denominator.

$$= \log(x^3y^5\sqrt{z}) - \log(wv^2)$$

The **multiplication property** allows you to break the log of $x^3y^5\sqrt{z}$ into three distinct logs and the log of wv^2 into two distinct logs.

$$= \log x^3 + \log y^5 + \log\sqrt{z} - (\log w + \log v^2)$$

Distribute the negative sign, and replace the square root sign with the exponent of ½ .

$$= \log x^3 + \log y^5 + \log z^{\frac{1}{2}} - \log w - \log v^2$$

Use the **power property** to change the exponents into numerical coefficients.

$$= 3\log x + 5\log y + \frac{1}{2}\log z - \log w - 2\log v$$

B. Use the properties for logarithms to rewrite 5 log x + 2 log (y-1) – log z – 1/3 log w as one logarithm.

$$5\log x + 2\log(y-1) - \log z - \frac{1}{3}\log w$$
$$= \log x^5 + \log(y-1)^2 - \log z - \log w^{\frac{1}{3}}$$

Begin by using the **power property** to change the numerical coefficients of the logs into exponents for the arguments. Do this for each log.

$$= \log x^5 + \log(y-1)^2 - \log z - \log \sqrt[3]{w}$$

Change fractional exponents into radicals.

$$= \log x^5 + \log(y-1)^2 - \left(\log z + \log \sqrt[3]{w}\right)$$

Factor out the negative sign.

$$= \log\left[x^5(y-1)^2\right] - \log\left(z\sqrt[3]{w}\right)$$

Becomes the numerator

Becomes the denominator

Use the **multiplication property** to make the sum of two logs into one log.

Use the **division property** to make the two remaining logs into one log.

$$= \log \frac{x^5(y-1)^2}{z\sqrt[3]{w}}$$

V. Using the inverse properties of logarithms, solve these equations

A. $\ln e^x = 7$

Use the inverse property $\ln e^x = x$.

Therefore $x = 7$.

B. $\log_6 36 = x$

$\log_6 36 = x$

36 can be written as

$\log_6 6^2 = x$

Use the inverse property $\log_b b^x = x$.

Therefore $x = 2$

C. $5^{\log_5 x} = 3$

Use the inverse property $b^{\log_b x} = x$.

Therefore $x = 3$.

D. $e^{\ln 4} = x$

Use the inverse property $e^{\ln x} = x$.

Therefore $x = 4$.

VI. Practice problems

1. Convert the logarithmic equation $\mathbf{\log_5 625 = 4}$ into an exponential equation.

2. Convert the exponential equation $\mathbf{2^5 = 32}$ into a logarithmic equation.

3. Solve the equation $\mathbf{3^{x+1} = 25}$ by using logarithms.

4. Solve the equation $\mathbf{17 = 4^{x-2}}$ by using logarithms.

5. Solve the equation $\mathbf{11^{x-1} = 2^x}$ by using logarithms.
   Hints: Take the log of both sides. $\log 11^{x-1} = \log 2^x$
   Use the power property to bring the exponents out as coefficients. (x-1) log11 = x log 2
   Distribute (x-1). x log 11 - log11 = x log 2
   Get all terms with x on the same side of the equation. x log 11- x log 2 = log 11
   Factor x out. x (log 11 – log 2) = log 11
   From this point, you are on your own! Use your basic math skills to isolate the x.

6. Use the properties for logarithms to rewrite $\log \frac{a^2\sqrt{b-c}}{d^5}$ as the sum or difference of logarithms.

   Hint: $\sqrt{b-c}$ can be written as $(b-c)^{\frac{1}{2}}$. <u>Remember that b-c is a binomial that cannot be made into separate logarithms.</u>

7. Use the properties for logarithms to rewrite 2 log (x + 3) + log (y – 1) − log (a + 4) – ½ log b as one logarithm.

8. Use the properties for logarithms to rewrite 1/3 log (a + 7) – ½ log b as one logarithm.

9. Solve for x

   a. $\text{Log}_{12} 144 = x$ b. $\ln e^{x+2} = 5$ c. $\mathbf{7^{\log_7 5} = x}$ d. $e^{\ln x} = 13$

10. Use the change of base formula and a calculator to evaluate these expressions.

    a. $\log_7 23$ b. $\log_2 81$ c. $\log_3 19212$ d. $\log_{11} 2003$

Answers

1. $5^4 = 625$
2. $\log_2 32 = 5$
3. $x = \dfrac{\log \frac{25}{3}}{\log 3} = 1.929947$
4. $x = \dfrac{\log 272}{\log 4} = 4.043731$
5. $x = \dfrac{\log 11}{\log\left(\frac{11}{2}\right)} = 1.406598$
6. 2 log a + ½ log (b – c) – 5 log d
7. $\log\left[\dfrac{(x+3)^2(y-1)}{(a+4)\sqrt{b}}\right]$
8. $\log \dfrac{\sqrt[3]{a+7}}{\sqrt{b}}$
9. a. x = 2
   b. x = 3
   c. x = 5
   d. x = 13
10. a. 1.61132528
    b. 6.339850003
    c. 8.977953792
    d. 3.17044761