Prince Sultan University
Department of Mathematical Sciences

Semester II, 2015 SPRING (Term 142)
May 23, 2015

MATH 111 – Calculus I
Final Exam

Time Allowed : 120 minutes
Maximum Points : 80 points

Name of the student: ____

ID number : ______________________________

Dr. Kamal Abodayeh	Dr. Nabil Mlaiki	Mr. Khaled Naseralla
Section 223	Section 225	Section 224
8 ---- 9	11 ----- 12	10 ----- 11

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination **has 11 problems** , some with several parts and a **total of 7 pages**. Make sure your paper has all these problems.

Question	Maximum score	Your Score
Q.1	18	
Q.2 , Q.3 , Q.4	13	
Q.5 , Q.6 , Q.7	12	
Q.8 , Q.9 , Q.10	23	
Q.11	14	
Total	80	

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<u>Q.1 (18 points):</u> Find the derivative, $\dfrac{dy}{dx}$. **(Simplify as much as possible)**

(i) $y = \left(\dfrac{x^2+1}{x^2-1}\right)^3$

(ii) $y = \dfrac{(x^2+5)\sin^4(2x)}{(x^3-8)^2}$

(iii) $y = \sqrt{x^2 \sin^{-1}(x^3)}$

(iv) $4x^2\cos(y) - 3\tan(x) = x^3 y^2$

(v) $y = \cosh^2\left(1+e^{3x}\right)$

(vi) $y = \dfrac{1}{x\sqrt{x^2+1}}$

Q.2 (4 points): Find the value of k at which the function is continuous at $x = 2$.

$$f(x) = \begin{cases} kx^2 + 4x + 1 & \text{if } x \le 2 \\ 6x^2 - 7k & \text{if } x > 2 \end{cases}$$

Q.3 (4 points): Find the point(s) at which the tangent line(s) to the graph of $y = 2x^3 - 8x + 1$ is(are) perpendicular to the line $2y - x + 2 = 0$?

Q.4 (5 points): Find the absolute maximum and minimum values of f on the given interval.

$$f(x) = 3x^4 + 4x^3 - 12x^2 + 1 \quad ; \; [-1,2]$$

Q.5 (4 points): Use the **Second Derivative Test** to find all the local maximum and minimum (if any) for the function: $y = \dfrac{x^2}{x-1}$

Q.6 (4 points): Verify that $f(x) = x^3 + x - 1$ satisfies the hypothesis of the **Mean Value Theorem** on the interval $[0, 2]$ and then find all the value(s) of c that satisfy the conclusion of the theorem.

Q.7 (4 points): A point (x, y) moves on the graph $y = x^3 + 2x^2 - 2x - 1$, such that the x coordinate is changing at a rate of 2 units/second. How fast is the y coordinate changing at the point $(-2, 3)$?

Q.8 (5 points): Find the point(s) on the curve $y = x^2$ that is (are) closest to the point $(0,1)$.

Q.9 (15 points): Evaluate the following limits. **(Show all your steps)**

a) $\lim_{x \to \infty} \left(\sqrt{x^2 + 4x + 1} - x \right)$

b) $\lim_{x \to 0} \frac{x^2 - \tan^{-1} x}{x \cos x}$

c) $\lim_{x \to 3} \frac{5x + 1}{x - 3}$

d) $\lim_{x\to 0}(\cos(x))^{\frac{1}{x^2}}$

e) $\lim_{x\to +\infty} x\left(e^{\frac{3}{x}}-1\right)$

<u>Q.10 (3 points):</u> Show that the function: $f(x)=2x^3+e^x-3$ has exactly one real zero.

Q.11 (14 points): Let $f(x) = x\, e^{-2x^2}$

(a) Find the domain of f .

(b) Determine the vertical and horizontal asymptotes, if any.

(c) Find the critical numbers and the intervals on which f is increasing and/or decreasing

(d) Find all the local maximum and/or local minimum, if any.

(e) Find the intervals on which f is concave up and/or concave down.

(f) Find the inflection point(s) of f , if any.

(g) Sketch the graph of f **showing all significant features.**