# Prince Sultan University

Deanship of Educational Services

Department of Mathematics and General Sciences

## COURSE DETAILS:

| **Business Calculus** | **MATH 211** | **FINAL EXAM** |
|---|---|---|
| Semester: | **Spring Semester --Term 172** | |
| Date: | **Monday May 14, 2018** | |
| Time Allowed: | 180 minutes | |

## STUDENT DETAILS:

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|---|---|
| **Student Name:** | |
| Student ID Number: | |
| Section: | **151** |
| Instructor's Name: | |

## INSTRUCTIONS:

- You may use a scientific calculator that does not have programming or graphing capabilities. NO borrowing calculators.
- NO talking or looking around during the examination.
- NO mobile phones. If your mobile is seen or heard, your exam will be taken immediately.
- Show all your work and be organized.
- You may use the back of the pages for extra space, but be sure to indicate that on the page with the problem.

## GRADING:

| **Page 1** | **Page 2** | **Page 3** | **Page 4** | **Page 5** | **Total** | **Total** |
|---|---|---|---|---|---|---|
| | | | | | | |
| **21** | **20** | **19** | **17** | **23** | **100** | **40** |

**Q.1 (6 points):** Find the following limits: **Show all your steps**

a) $\lim_{x\to+\infty} \dfrac{1-3x^3}{2x^3+6x+2}$

b) $\lim_{x\to 1} \dfrac{x^2+4x-5}{x^2-1}$

**Q.2 (5 points):** Determine whether $f(x)=\begin{cases} -x^2+2x-3 & \text{if } x\geq 4 \\ 6x-3 & \text{if } 2<x<4 \\ 3-x+2x^2 & \text{if } x\leq 2 \end{cases}$ is **continuous** at $x=2$. **Show all steps**

**Q.3 (5 points):** Find the **equation of the tangent line** to the curve $y\cdot\ln x+y^2=x$ at $(1,-1)$

**Q.4 (5 points):** $\$50,000$ was deposited in a bank account. The money was tripled 12 years later. **What is the interest rate** if it is compounded continuously?

**Q.5 (9 points):** Find the derivative, $\frac{dy}{dx}$. **Simplify reasonably.**

a) $y = \sqrt{\frac{1-2x}{3x+2}}$

b) $y = \ln\sqrt{2x^2+3}$

c) $y = x^2e^{-x}(3x+5)^3$

**Q.6 (6 points):** A manufacturer estimates when $x$ units of a certain commodity are produced each month, the total cost will be $C(x) = x^3 + 5x + 162$ dollars, and all $x$ units can be sold at the price $p(x) = 180 - 2x$ dollars per unit. Determine the **level of production,** $x$ that results in **maximum profit.**

**Q.7 (5 points):** A manufacturer's total cost is $C(q) = 0.1q^3 - 0.5q^2 + 500q + 200$ dollars when $q$ thousand units are produced. Currently 4 thousand units ($q = 4$) are being produced and the manufacturer is planning to increase the level of production to 4,300 units. Use marginal analysis to estimate how this change will affect the total cost.

**Q.8 (13 points):** Given the function $f(x)=\frac{x}{x^2+1}$ **DO NOT GRAPH** **Show all your steps**

a) Find the vertical asymptotes, if any.

b) Find the horizontal asymptote, if any.

c) Find the critical points, if any.

d) Find the intervals of increase and decrease, if any.

e) Find the relative maximum and minimum, if any.

f) Find the intervals of concave up and concave down, if any.

g) Find the inflection points, if any.

**Q.9 (6 points):** A certain machine loses from its value with time so it's value after $t$ years becomes $Q(t)=20,000e^{-0.4t}$ dollars.

a) At what rate is the value of the machine changing with respect to time after 5 years? Is the value increasing or decreasing with time?

b) At what percentage rate is the value of the machine changing with respect to time after 5 years?

**Q.10 (11 points):** Evaluate the integrals:

a) $\int_1^6 x^2(x-1)dx$

b) $\int x \cdot e^{-5x} dx$

c) $\int_0^1 \frac{e^{2x} dx}{\sqrt{1+e^{2x}}}$

**Q.11 (6 points):** Saudi Airlines determines that when a round trip ticket between Riyadh and Jeddah costs $p$ Riyals, the daily demand for the tickets is $q = 256 - 0.001p^2$

a) Find the **elasticity of demand** function.

b) If the Airline is currently charging 300 Riyals for the ticket, do you recommend that they **raise or lower this price** based on the elasticity of demand from part (a)? **Explain.**

**Q.12 (6 points):** Find the area of the region bounded by the curves: $y = x^2 - 4$ and $y = 2x - 1$

**Q.13 (6 points):** The output of a factory is changing at the rate $Q'(t) = 2t^3 - 3t^2 + 10t + 3$ units/hour, where t is the number of hours after the morning shift begins at 8 A.M. **How many units** are produced between 10 A.M. and noon?

**Q.14 (5 points):** A manufacturer supplies $S(p) = 0.5p^2 + 3p + 7$ **hundred units** of a certain commodity to the market when the price is $p$ dollars per unit. Find the **average supply** as the price varies from $p$ = \$2 to $p$ = \$5.

**Q.15 (6 points):** Compute the partial derivatives, $f_x$, $f_y$ $f_{xx}$, $f_{yy}$, $f_{xy}$, and $f_{yx}$.

$$f(x, y) = 5x^2y + e^{xy} + 3$$