Prince Sultan University

Department of Mathematical Sciences

Semester II, 2014 SPRING (Term 132)
May 24, 2014

MATH 111 – Calculus I
Final Exam

Time Allowed : 120 minutes
Maximum Points : 100 points

Name of the student: _____

ID number : _____

Dr. Nasreddine Megraz	Mr. Abid Zargar	Mr. Khaled Naseralla
Section 219	Section 218	Section 217
11 ---- 12	10 ----- 11	8 ----- 9

Important Instructions:

1. You may use a scientific calculator that does not have programming or graphing capabilities.
2. You may NOT borrow a calculator from anyone.
3. You may NOT use notes or any textbook.
4. There should be NO talking during the examination.
5. Your exam will be taken immediately if your mobile phone is seen or heard
6. Looking around or making an attempt to cheat will result in your exam being cancelled
7. This examination **has 12 problems** , some with several parts and a **total of 6 pages**. Make sure your paper has all these problems.

Question	Maximum score	Your Score
Q.1 , Q.2	24	
Q.3 , Q.4 , Q.5 , Q.6	21	
Q.7	18	
Q.8 , Q.9 , Q.10	18	
Q.11 , Q.12	19	
Total	100	

40

Q.1 (4 points): Find a formula for the inverse function, $f^{-1}(x)$ for $f(x)=\dfrac{e^x}{1+2e^x}$

Q.2 (20 points): Find the derivative, $\dfrac{dy}{dx}$. **(Simplify as much as possible)**

(i) $y=e^{3x}\left[\cos(2x)\right]^3$

(ii) $y=\left(\dfrac{\sqrt{x}.\sqrt[3]{x+1}}{\sin(x).\sec(x)}\right)$

(iii) $y=x^3.\ln\left(\cos^{-1}(x^3)\right)$

(iv) $x^3y^2-3y\sin(x)=x.e^{3y}$

(v) $y=\sqrt{\tan^{-1}(3^{2x})}$

Q.3 (6 points): Find the values of a and b so that the following function is continuous at $x=2$

$$f(x)=\begin{cases} x^2+ax+2 & x<2 \\ 5+b & x=2 \\ bx+4 & x>2 \end{cases}$$

Q.4 (5 points): Find the equation of the tangent line to the graph of $y=x^2\sqrt{5-x^2}$ at the point where $x=1$

Q.5 (6 points): Find the absolute maximum and minimum values of $f(x)=\left(x^2-4x\right)^{\frac{2}{3}}$ on the closed interval $[-2,3]$

Q.6 (4 points): Use the **Second Derivative Test** to determine all local maximum and minimum points for $f(x)=1+10x^2-5x^3$

Q.7 (18 points): Evaluate the following limits:

a) $\lim_{x\to 1} \dfrac{x^3 - 3x^2 + 2x}{x-1}$

b) $\lim_{x\to 0} \dfrac{3x - \sin(kx)}{x}$, $k \neq 0$

c) $\lim_{x\to \infty} \left(1 - \dfrac{2}{x}\right)^x$

d) $\lim_{x\to 0} \dfrac{e^{4x} - 1 - 4x}{x^2}$

e) $\lim_{x\to 9} \dfrac{\sqrt{x}}{(x-9)^3}$

Q.8 (7 points): Let $f(x) = \dfrac{x^2 - x}{x^2 - 3x + 2}$

a) Find the domain of f.
b) Determine the vertical and horizontal asymptotes, if any.(show your work)
c) Find all the critical numbers.
d) Find the intervals on which f is increasing and/or decreasing.
e) Find the local maxima and/or local minima, if any.

Q.9 (5 points): Two ships leave port at noon. One ship sails north at $6 miles/hour$ and the other sails east* at $8 miles/hour$. At what rate is the distance between the two ships increasing $2 hours$ later?

Q.10 (6 points): A cylindrical can, open at the top, is to hold 1000 cm^3 of liquid. Find the height and radius that minimize the amount of material needed to manufacture the can.

Q.11 (5 points): Verify that $f(x) = \frac{1}{1-x}$ satisfies the hypothesis of the Mean Value Theorem on the interval $[3,4]$, then find the value(s) of c that satisfy the conclusion of the theorem.

Q.12 (14 points): Let $f(x) = x^4 + 8x^3 + 10$

(a) Determine the vertical and horizontal asymptotes, if any.

(b) Find the critical numbers and the intervals on which f is increasing and/or decreasing

(c) Find the intervals on which f is concave up and/or concave down.

(d) Find the inflection point(s) of f, if any.

(e) Sketch the graph of f **showing all significant features.**