Prince Sultan University

Department of Mathematical Sciences

Final Exam

Semester I, 2005/2006 Fall (051)

26th January, 2006

MATH 111 – CALCULUS I

Mr. Khaled Naseralla

Time Allowed : 3 hours
Maximum Points: 100 points

Name of the student : _____

ID number : _____

Section : _____

For All The Students:

- **Answer all the questions.**
- **This exam consists of *a total of 8 pages and 14 questions.***
- **Show your working in the space provided for each question.**
- **Show all the key steps of your work.**
- **Scientific, non-programmable calculators are allowed.**

Question	*Maximum score*	*Your Score*
Q.1	**5**	
Q.2	**5**	
Q.3	**6**	
Q.4	**5**	
Q.5	**6**	
Q.6	**12**	
Q.7	**9**	
Q.8	**5**	
Q.9	**5**	
Q.10	**12**	
Q.11	**8**	
Q.12	**7**	
Q.13	**5**	
Q.14	**10**	
Total	***100***	

Q.1: **Find the natural domain of** $f(x)=\sqrt{\dfrac{x-1}{x^2-9}}$ **.** **(5 points)**

Q.2: **For what values of** x **is the following function *discontinuous*?** **(5 points)**

$$f(x)=\begin{cases}\dfrac{x-1}{\sqrt{x}-1} & x>1\\ 5-3x & -2\le x\le 1\\ \dfrac{6}{x-4} & x<-2\end{cases}$$

Q.3: **Find an equation of the line perpendicular to** $y-2x=4$ **and passing through the point** $(1,2)$ **.** **(6 points)**

***Q.4*:** **Consider the function** $f(x)=3x^2-2x$ **over the interval** $[2,2.5]$**.** **(5 points)**

a) Compute dy **and** Δy **for the given interval.**

b) Compute the average rate of change of f **on the given interval.**

Q.5: **Given that** $f(0)=2$ **,** $g(0)=3$ **,** $f'(0)=5$ **,** $g'(0)=7$ **, and** $f'(3)=\prod$ **.** **(6 points)**

Compute:

a) $h'(0)$ **if** $h(x)=f(x)g(x)$

b) $k'(0)$ **if** $k(x)=(f\circ g)(x)$

Q.6: **Find the limits:** **(12 points)**

a) $\lim\limits_{x\to 3^-}\dfrac{x}{x-3}$

b) $\lim\limits_{x\to 0}\dfrac{\sin 3x}{5x}$

c) $\lim\limits_{x\to 3}\dfrac{\sqrt{3x}-x}{x^2-9}$

d) $\lim\limits_{x\to -4}\dfrac{2x+8}{x^2+x-12}$

e) $\lim_{x\to-\infty}\dfrac{\sqrt{5x^2-2}}{x+3}$

f) $\lim_{x\to\infty}\dfrac{5x+15x^7}{10x^9-4x}$

Q.7: **Consider the function** $f(x)=\begin{cases}\dfrac{1}{x^2} & \mathbf{x<-1}\\ 2 & -1\le x<1\\ 3 & x=1\\ x+1 & 1<x\le 2\\ \dfrac{-1}{(x-2)^2} & \mathbf{x>2}\end{cases}$ **(9 points)**

Determine the following limits:

a) $\lim_{x\to-1^+} f(x)=$

b) $\lim_{x\to-1^-} f(x)=$

c) $\lim_{x\to-1} f(x)=$

d) $\lim_{x\to1} f(x)=$

e) $\lim_{x\to-3} f(x)=$

f) $\lim_{x\to5} f(x)=$

Q.8: **Find the equation of the tangent line to the curve of the graph of** $x^2y^2-2x=4-4y$ **at** $(2,-2)$ **(5 points)**

Q.9: **Consider the function** $f(x)=\begin{cases}a+bx & x>2\\ 7 & x=2\\ b+ax^2 & x<2\end{cases}$ **(5 points)**

Determine the values of the constants a **and** b **that will make the function** $f(x)$ **continuous everywhere.**

Q.10 **: Find** $\dfrac{dy}{dx}$ **of each of the following. Simplify your answer** **(12 points)**

a) $y=\dfrac{x^2\sin^2 x}{(x+1)(x+2)^2}$

b) $y = \dfrac{\tan 2x^2}{(2x+1)^2}$

c) $y = x^{\sqrt{x}} e^{3x+4}$

d) $y = \sqrt{x}\,\tan^{-1} 5x$

e) $\dfrac{x - y^3}{y + x^2} = x + 2$

f) $y = \ln(\cos^5(3x^4))$

***Q.11*:** **Consider the function $f(x) = x^6 - 2x^3$ on the interval $[-2, 2]$.** **(8 points)**

a) Find the $x-$ and $y-coordinates$ of all the relative extrema and classify each as a relative maximum or a relative minimum.

b) Find the $x-$ and $y-coordinates$ of all the absolute extrema and classify each as an absolute maximum or an absolute minimum.

c) Find the $x-coordinates$ of all the inflection points.

d) How would the above answers change if the domain of $f(x)$ were all real numbers.

***Q.12*:** **The position function of a particle moving along a coordinate line is given by** **(7 points)**
$s(t) = t^3 - 6t^2 + 1$ **,** $t \geq 0$ **where s is in meters(m) and t is in seconds(s)**

a) At what time is the particle stopped?
b) When is the particle speeding up? Slowing down?
c) Find the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

***Q.13*:** **A box with a square base and with no top has a surface area of $108m^2$.** **(5 points)**
Find its dimensions that will maximize the volume.

***Q.14*:** **Let $f(x) = x^3 - 3x^2 - 24x$, find** **(10 points)**

a) The intervals on which $f(x)$ is increasing.
b) The intervals on which $f(x)$ is decreasing.
c) The intervals on which $f(x)$ is concave up.
d) The intervals on which $f(x)$ is concave down.
e) Give a rough sketch of $f(x)$ showing the relative maxima and minima and the inflection points.

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